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Subspace based system identification for an acoustic enclosure

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Abstract

This paper is aimed at identifying a dynamical model of an acoustic enclosure, a duct with rectangular cross section, closed ends, and side mounted speaker enclosures. Acoustic enclosures are known to be resonant systems of high order. In order to design a high performance feedback controller for a duct or an acoustic enclosure, one needs to have an accurate model of the system. Subspace based system identification techniques have proven to be an efficient means for identifying dynamics of high order highly resonant systems such as flexible structures. In this paper a frequency domain subspace based method together with a second iterative optimization step minimizing a frequency domain least-squares criterion is successfully employed.

1 Introduction

During the previous decade there has been a substantial and consistent focus on the problem of active noise cancellation (ANC) in acoustic enclosures. Early work concentrated mainly on adaptive feedforward configurations such as those detailed in [7]. Such techniques involve the measurement of a disturbance and attempt to arrest the propagation downstream. Although impressive results have been achieved for ducts with end mounted disturbances [14, 15] new approaches have been required to confront the greater problems of multiple disturbances, three dimensional sound fields, and spatial cancellation. More recent work involves the design of feedback control systems to cancel or absorb noise in acoustic enclosures [1].

Feedback control strategies require a model of the system to be controlled. Analytic modeling often results in a poor model if the system is even mildly realistic (refer [4]). Generally system identification methods are employed for this purpose.Such methods are convenient as they model the overall system i.e. the acoustic dynamics of the duct as well exterior systems such as actuator dynamics, amplifiers, and filters. These overall compound models are required for control system design.

Methods which identify state-space models by means of geometrical properties of the input and output sequences are commonly known as subspace methods and have received much attention in the literature (see [19] for a survey of time domain techniques). One of the advantages with subspace methods is that an estimate is calculated without any non-linear parametric optimization. In classical prediction error minimization [9], such a step is necessary for most model structures. A second advantage is that the identification of multivariable systems is just as simple as for scalar systems.

In this paper we consider the case when data is given in the frequency domain, i.e. when samples of the Fourier transform of the input and output signals are taken as the primary measurements. In a number of applications, particularly when modeling flexible structures, it is common to fit models in the frequency domain [16, 13]. A few subspace based algorithms formulated in the frequency domain has appeared recently [8, 11, 17, 10]

From a statistical point of view it is well known that, under some assumptions, the best models are obtained by the method of maximum-likelihood. In this paper we will, as a second step, after obtaining an initial model from the subspace method, invoke a parametric optimization minimizing the 2-norm of the frequency domain error. Under suitable assumptions this can be interpreted as a maximum-likelihood estimation step [13]. It is important to point out that the success of the second parametric optimization is heavily based on the availability of a good starting point for the optimization.

Preliminaries

Consider a stable time-invariant discrete time linear system of finite order n in state-space form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) + v(k) \end{aligned} \qquad (1)$$

where $u(k) \in \mathbb{R}^m$ is the input vector, $y(k) \in \mathbb{R}^p$ the output vector and $x(k) \in \mathbb{R}^n$ is the state vector. By considering real valued signals we implicitly assume that the matrix quadruple (A, B, C, D) is also real valued. The noise term $v(k) \in \mathbb{R}^p$ is assumed to be independent of the input sequence $u(t)$. Here the time

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index k denotes normalized time. Hence $y(k)$ denotes the sample of the output signal $y(t)$ at time instant $t = kT$ where T denotes the sample time. We also assume that the state-space realization (1) is minimal which implies both observability and controllability [6]. A system with this type of noise model is commonly known as output-error models [9]. Note that all such pairs (1) describing the same input/output behavior of the system are equivalent under a non-singular similarity transformation $T \in \mathbb{R}^{n \times n}$ [6], *i.e.* the matrices $(T^{-1}AT, T^{-1}B, CT, D)$ will be a state-space realization with equivalent i/o properties. The discrete time Fourier transform $\mathcal{F}$ of a sequence $f(k)$ is defined as

$$\mathcal{F}f(k) = F(\omega) = \sum_{k=-\infty}^{\infty} f(k)e^{-j\omega k} \tag{2}$$

where $j = \sqrt{-1}$. Applying the Fourier transform to (1) gives

$$\begin{aligned} e^{j\omega}X(\omega) &= AX(\omega) + BU(\omega) \\ Y(\omega) &= CX(\omega) + DU(\omega) + V(\omega) \end{aligned} \tag{3}$$

where $Y(\omega), U(\omega), V(\omega)$ and $X(\omega)$ are the transformed output, input, noise and state respectively. By eliminating the state from (3) we obtain

$$Y(\omega) = G(e^{j\omega})U(\omega) + V(\omega) \tag{4}$$

where $G(z) = D + C(zI - A)^{-1}B$ is known as the transfer function of the linear system.

The Identification Problem

Given samples of the discrete time Fourier transform of the input signal $U(\omega)$ and output signal $Y(\omega)$ at N arbitrary frequency points ω_k; find a state-space model of the form (1) which well approximates the data in a least-squares fashion, i.e.

$$\hat{G}(z) = \arg\min_{G(z)} \sum_{k=1}^{N} \left\| Y(\omega_k) - G(e^{j\omega_k})U(\omega_k) \right\|^2 \tag{5}$$

2 Identification method

This section is devoted to describing the identification technique used. As a first step a state-space model is identified using a frequency domain subspace based algorithm. The identified state-space realization is then transformed to a tridiagonal realization suitable for a tridiagonal parametrization. Finally a minimization is employed which minimize the sum of the 2-norm of the identification error (see (5)). Here the tridiagonal parametrization and an iterative Gauss-Newton nonlinear least-squares algorithm is utilized to find a (local) optimum of the least-squares criterion function.

Frequency domain subspace method

In this section we will derive the basic relations which the subspace based identification algorithm rely upon and present a simple but yet powerful identification algorithm. Let us introduce the vector

$$W(\omega) = \begin{bmatrix} 1 & e^{j\omega} & e^{j2\omega} & \cdots & e^{j\omega(q-1)} \end{bmatrix}^T \tag{6}$$

the extended observability matrix with q block rows

$$\mathcal{O}_q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{q-1} \end{bmatrix} \tag{7}$$

and the lower triangular Toeplitz matrix

$$\Gamma_q = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-2}B & CA^{q-3}B & \cdots & D \end{bmatrix} \tag{8}$$

By recursive use of (3) we obtain

$$\begin{aligned} W(\omega) \otimes Y(\omega) = &\mathcal{O}_q X(\omega) + \Gamma_q W(\omega) \otimes U(\omega) \\ &+ W(\omega) \otimes V(\omega) \end{aligned} \tag{9}$$

where $\otimes$ denotes the standard Kronecker product [6]. The extended observability matrix $\mathcal{O}_q$ has a rank equal to the system order n if $q \geq n$ since the system (A, B, C, D) is minimal.

If N samples of the transforms are known we can collect all data into one complex matrix equation. Define the diagonalization operator for a sequence of vectors z_i of length p as

$$\operatorname{diag}[z_1, z_2, \ldots, z_N] \triangleq \begin{bmatrix} z_1 & 0 & & 0 \\ 0 & z_2 & \ddots & \\ & \ddots & \ddots & \\ 0 & & 0 & z_N \end{bmatrix} \tag{10}$$

which is a tall (or square) matrix of size $Np \times N$. By introducing the additional matrices

$$\begin{aligned} \mathbf{W}_{N,p} &= \begin{bmatrix} W(\omega_1) & W(\omega_2) & \cdots & W(\omega_N) \end{bmatrix} \otimes I_p \\ \mathbf{Y}^c &= \frac{1}{\sqrt{N}} \mathbf{W}_{N,p} \operatorname{diag}[Y(\omega_1), \ldots, Y(\omega_N)] \in \mathbb{C}^{qp \times N}, \\ \mathbf{U}^c &= \frac{1}{\sqrt{N}} \mathbf{W}_{N,m} \operatorname{diag}[U(\omega_1), \ldots, U(\omega_N)] \in \mathbb{C}^{qm \times N}, \\ \mathbf{V}^c &= \frac{1}{\sqrt{N}} \mathbf{W}_{N,p} \operatorname{diag}[V(\omega_1), \ldots, V(\omega_N)] \in \mathbb{C}^{qp \times N}, \\ \mathbf{X}^c &= \frac{1}{\sqrt{N}} \begin{bmatrix} X(\omega_1), & \cdots, & X(\omega_N), \end{bmatrix} \in \mathbb{C}^{n \times N}, \end{aligned}$$

and using (9) we arrive at the matrix equation

$$\mathbf{Y}^c = \mathcal{O}_q \mathbf{X}^c + \Gamma_q \mathbf{U}^c + \mathbf{V}^c. \tag{11}$$

The superscript c is used to stress that the matrix is complex valued. Clearly since the system matrices are assumed real valued $\mathcal{O}_q$ and Γ_q are also real. Hence by forming a real matrix from the real and complex parts of $\mathbf{Y}^c$ as

$$\mathbf{Y} = [\ \mathrm{Re}\{\mathbf{Y}^c\} \quad \mathrm{Im}\{\mathbf{Y}^c\}\ ]$$

and similarly for $\mathbf{U}, \mathbf{X}, \mathbf{V}$ we obtain the real valued matrix equation

$$\mathbf{Y} = \mathcal{O}_q \mathbf{X} + \Gamma_q \mathbf{U} + \mathbf{V}. \tag{12}$$

Note that this equation now has $2N$ columns. As the number of frequency samples increases the number of columns in the matrix equation also increases. The normalization with $\frac{1}{\sqrt{N}}$ ensures that the norm of the matrix stays bounded as the number of frequencies (columns) tends to infinity. The number of (block)-rows q is up to the user to choose but must be larger than the upper bound of the model orders which will be considered.

The identification scheme we employ to find an state-space model $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ is based on a two step procedure. First the relation (11) is used to consistently determine a matrix $\hat{\mathcal{O}}_q$ with a range space equal to the extended observability matrix $\mathcal{O}_q$. From $\hat{\mathcal{O}}_q$ it is straightforward to derive $\hat{A}$ and $\hat{C}$ as is well known from the time domain subspace methods [19]. In the second step $\hat{B}$ and $\hat{D}$ are determined by minimizing the 2-norm of the error

$$\hat{B}, \hat{D} = \arg\min_{B,D} V_N(B, D) \tag{13}$$

$$V_N(B, D) = \sum_{k=1}^{N} \left\| Y(\omega_k) - \hat{Y}(\omega_k, B, D) U(\omega_k) \right\|^2 \tag{14}$$

where

$$\hat{Y}(\omega_k, B, D) = \left[D - \hat{C}(e^{j\omega_k} I - \hat{A})^{-1} B \right] U(\omega_k)$$

which has an analytical solution since the output $\hat{Y}$ is a linear function of both B and D assuming $\hat{A}$ and $\hat{C}$ are fixed. Hence once $\hat{A}$ and $\hat{C}$ are derived $\hat{B}$ and $\hat{D}$ is determined by linear regression.

The Basic Projection Method

The first step of the subspace method aims at providing an estimate of the range space of the observability matrix. First consider the noise free case $\mathbf{V}_{q,N} = 0$ and we restate the basic projection method in the frequency domain. In (12) the term $\Gamma_q \mathbf{U}$ can be removed by a projection. Denote by $\Pi_N^{\perp}$ the orthogonal projection onto the null-space of $\mathbf{U}$,

$$\Pi^{\perp} = I - \mathbf{U}^T (\mathbf{U}\mathbf{U}^T)^{-1} \mathbf{U} \tag{15}$$

here $\mathbf{U}^T$ denotes the transpose of the matrix $\mathbf{U}$. The inverse in (15) will exist if the system is sufficiently excited by the input. Since

$$\mathbf{U}\Pi^{\perp} = 0$$

the effect of the input will be removed and we obtain

$$\mathbf{Y}\Pi^{\perp} = \mathcal{O}_q \mathbf{X}\Pi^{\perp}. \tag{16}$$

Provided

$$\mathrm{rank}\,(\mathbf{X}\Pi^{\perp}) = n \tag{17}$$

$\mathbf{Y}\Pi^{\perp}$ and $\mathcal{O}_q$ will span the same column space. The mild conditions required for (17) to hold can be found in [11].

A matrix which concisely span the column space of $\mathbf{Y}_{q,N}\Pi^{\perp}$ can be recovered in a singular value decomposition [5]

$$\mathbf{Y}\Pi^{\perp} = [\ U_s \quad U_o\ ] \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_o \end{bmatrix} \begin{bmatrix} V_s^T \\ V_o^T \end{bmatrix} \tag{18}$$

where $U_s \in \mathbb{R}^{qp \times n}$ contains the n principal singular vectors and the diagonal matrix Σ_s the corresponding singular values. In the noise free case $\Sigma_o = 0$ and there will exist a nonsingular matrix $T \in \mathbb{R}^{n \times n}$ such that

$$\mathcal{O}_q = U_s T.$$

This shows that U_s is an extended observability matrix $\hat{\mathcal{O}}_q$ of the original transfer function for some realization. By the shift structure of the observability matrix (7) we can proceed to calculate A and C as

$$\hat{A} = \arg\min_{A} \|J_1 U_s A - J_2 U_s\|_F^2 = (J_1 U_s)^{\dagger} J_2 U_s \tag{19}$$

$$\hat{C} = J_3 U_s \tag{20}$$

where J_i are the selection matrices defined by

$$J_1 = (\ I_{(q-1)p} \quad 0_{(q-1)p \times p}\ ), \tag{21}$$

$$J_2 = (\ 0_{(q-1)p \times p} \quad I_{(q-1)p}\ ), \tag{22}$$

$$J_3 = (\ I_p \quad 0_{p \times (q-1)p}\ ) \tag{23}$$

and I_i denotes the $i \times i$ identity matrix, $0_{i \times j}$ denotes the $i \times j$ zero matrix, $\|\cdot\|_F$ is the Frobenius norm and $X^{\dagger} = (X^T X)^{-1} X^T$ denotes the Moore-Penrose pseudo-inverse of the full rank matrix X. With the knowledge of $\hat{A}$ and $\hat{C}$, $\hat{B}$ and $\hat{D}$ are easily determined from (13).

Effective Implementation

A most effective way of forming the matrix $\mathbf{Y}\Pi^{\perp}$ is by use of the QR factorization of the matrix, (see [18])

$$\begin{pmatrix} \mathbf{U} \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} \mathbf{R}_{11} & 0 \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_1^T \\ \mathbf{Q}_2^T \end{pmatrix}. \tag{24}$$

Straightforward calculations reveal that $\mathbf{Y}\Pi^{\perp} = \mathbf{R}_{22}\mathbf{Q}_2^T$ and the column space of $\mathbf{R}_{22}$ is equal to the column space of $\mathbf{Y}\Pi^{\perp}$ and it suffices to use $\mathbf{R}_{22}$ in the SVD (18).

Consistency Issues

As we have seen the basic projection algorithm will estimate a state-space model which is similar to the original realization in the noise free case. If we now let the noise term $V(\omega)$ be a zero mean complex random variable the issue of consistency becomes important. Does the estimate converge to the true system as N, the number of data, tends to infinity? Consistency of the basic projection algorithm and the related algorithm [8] has been investigated in [11, 17]. The result is that unless the covariance structure of the data is known, consistency cannot be expected. However, for the application studied in this paper the basic projection method produces sufficiently good estimates although the noise structure is unknown. A second approach based on the classical instrumental variable technique (IV) [9] does not require knowledge of the variance properties (or equivalently the color of the noise). Subspace based time IV-techniques can be found in [18]. A frequency domain subspace-IV-approach can be found in [10].

Non-linear optimization

The initial estimate delivered by the subspace method is a good starting point for a non-linear iterative optimization of the desired criterion (5). In order to successfully do so, particularly for high order systems, a numerically sound parametrization of the transfer function $G(z)$ is required. A parametrization based on the state-space form (1) using a tridiagonal A matrix has recently been suggested [12]. Particularly the tridiagonal parametrization has shown promising numerical properties for high order systems. Let

$$G(z,\theta) = D(\theta) + C(\theta)(zI - A(\theta))^{-1}B(\theta) \qquad (25)$$

be a transfer function parametrized through the state-space matrices by the real valued vector θ using the tridiagonal model structure. A Gauss-Newton algorithm [3] is employed for finding a solution to the parametric optimization problem

$$\hat{\theta} = \arg\min_{\theta} \sum_{k=1}^{N} \left\| Y(\omega_k) - G(e^{j\omega_k},\theta)U(\omega_k) \right\|^2 \qquad (26)$$

The optimization algorithm is started from the parameter vector θ_0 representing the transfer function delivered by the subspace method. For high order systems it is vital to have a high quality starting point in order to converge to a good optimum. Since the Gauss-Newton algorithm only converges locally to a minimum a starting point close to the global optimum is desirable. If we assume the frequency domain noise is complex Gaussian and white the solution to (26) will be the maximum-likelihood estimate [13].

3 The Experiment

A plan view of the acoustic duct apparatus is shown in Figure 1. For disturbance rejection a feedback loop is to be closed around the control speaker to microphone path. This path represents a system where the speaker signal and microphone voltage are the input and output respectively. The system dynamics consists of a series combination of the following transfer functions: speaker signal low pass filter, power amplifier, applied speaker voltage to baffle acceleration, baffle acceleration to sound pressure, and the gain of the microphone from sound pressure to voltage.

Speaker 2 is used in control system experimentation and is redundant during identification. Although it is unused, it must remain attached. This is due to the coupling that exists between the passive dynamics of the speaker and the enclosed sound field. (See [2] for a discussion of the coupling between passive speaker dynamics and enclosed sound fields.)

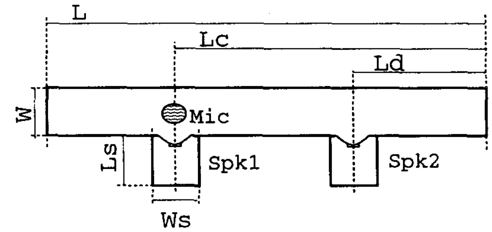

Figure 1: Plan view of the acoustic duct apparatus.

Dimension	Value
L	4.840 m
Ls	0.320 m
Lc	2.940 m
Ld	0.940 m
W	0.246 m
Ws	0.246 m
Hwight	0.295 m

Table 1: Duct Internal Dimensions

The acoustic actuators are constructed from 10 inch diameter speakers (Jaycar CS-2220) and a sealed enclosure of 23 liters. The frequency response between applied voltage and baffle acceleration was measured to determine a low frequency bandwidth of 55 Hz. The measurement was taken using a Polytec scanning laser vibrometer (PSV-300) and is shown in Figure 2.

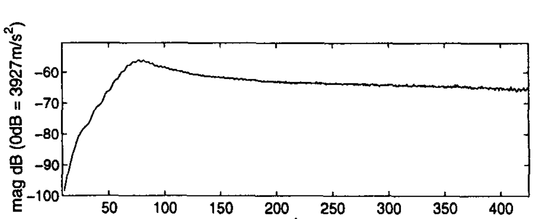

Figure 2: Magnitude response of the acoustic actuators from applied voltage to baffle acceleration

The acoustic sensor is a unidirectional dynamic microphone (Schure SM58) with a bandwidth of 50 to 15,000 Hz and a pressure sensitivity of -56 dB (0 dB = $1V/\mu bar$).

The excitation signal generation and data recording is performed using the dSPACE rapid prototyping system. The excitation signal is a uniformly distributed random process sampled at 2 kHz and digitally filtered with a 6th order elliptic bandpass filter between 50 and 400 Hz. The DAC output is filtered with a 1 kHz analog low pass filter to remove the sampling frequency component and harmonics. Input/output data is filtered using lowpass 1 kHz 4th order anti-aliasing filters and is recorded at 2 kHz with 16 bit resolution for 60 seconds.

4 Identification results

The experimental input-output time series was divided into two equal datasets, one estimation set and one set which we set aside for validation purposes. The estimation data is transformed to the frequency domain by use of the fast Fourier transform (FFT) without any windowing functions. As the frequency content of the excitation signal is restricted to the 50-400 Hz frequency band all frequency data outside this interval is discarded. This reduces the size of the data sets to 5456 samples. As the input to the system is based on a filtered random signal the spectrum of the input fluctuates over the frequencies. In order to only use highly excited frequencies, only frequency points with an input amplitude above 1.7 are retained in the identification set. This leads to an identification set with 1545 points. The subspace estimation algorithm is then employed to estimate a model of order 29 using $q = 60$ as the number of rows in the matrix equation (12).

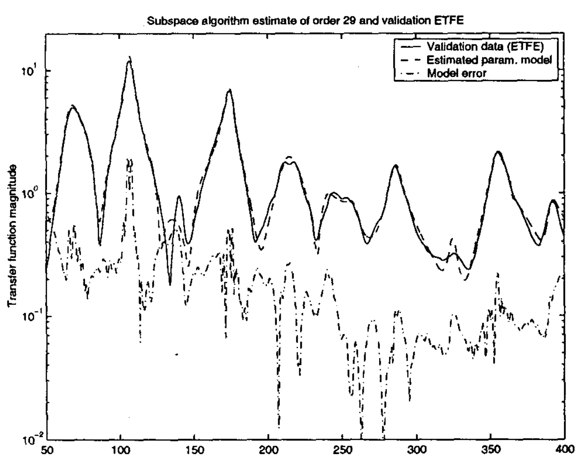

Figure 3: Transfer functions of parametric model estimate of order 29 from subspace algorithm and ETFE from validation data. Also the magnitude of the error is shown.

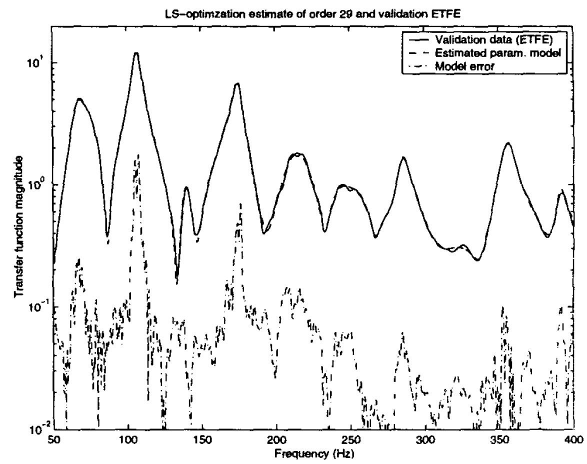

Figure 4: Final estimated model after parametric optimization of LS criterion. Amplitude plot and error plot showing estimated model of order 29 and ETFE from validation data.

To validate the model, or rather to check if there is any evidence in the data which implies that the model is *invalid*, we use the independent validation data set to derive the Empirical Transfer Function Estimate (ETFE) [9] of the system. In particular the ETFE is calculated at $N_v = 2^{14} = 16384$ frequency points equally spaced between 0 and 1kHz. The ETFE is the fraction between the cross spectrum between the input and output and the auto spectrum of the input.

$$\hat{H}(\omega) = \frac{\hat{\Phi}_{yu}(\omega)}{\hat{\Phi}_{uu}(\omega)} \tag{27}$$

In the calculation of the spectral estimates $\hat{\Phi}_{yu}(\omega)$ and $\hat{\Phi}_{uu}(\omega)$ a Hamming window of size 512 is employed to smooth the estimates.

The transfer functions of the subspace estimate and the ETFE are plotted together in Figure 3. The RMS error between the parametric model $\hat{G}$ and the validation ETFE $\hat{H}$ is 0.28. The RMS error is defined as

$$\text{RMS} = \sqrt{\frac{1}{N_v}\sum_{k=1}^{N_v}\left\|\hat{H}(\omega_k) - \hat{G}(e^{j\omega_k})\right\|^2} \tag{28}$$

The nonlinear optimization step further improved the RMS error to a value of 0.17 again calculated against the validation data ETFE. The amplitude of the finally estimated transfer function is depicted in Figure 4 together with the validation data ETFE. Also the magnitude of the complex error between the estimated model and the validation data is shown in the figure.

As a second validation step a time domain simulation comparing the output of the model to the time domain validation output. The validation input is used to simulate the output of the 29th order model. An excerpt of the simulation is shown in Figure 5. The output of the model is almost identical to the measured output of the validation data.

Based on the cross validation both in the frequency domain as well as the time domain it can be concluded that the estimated model is quite accurate. Hence the model should be a good candidate for use in a model based control design although bearing in mind the level of estimation error or uncertainty as indicated by the error curve in Figure 4.

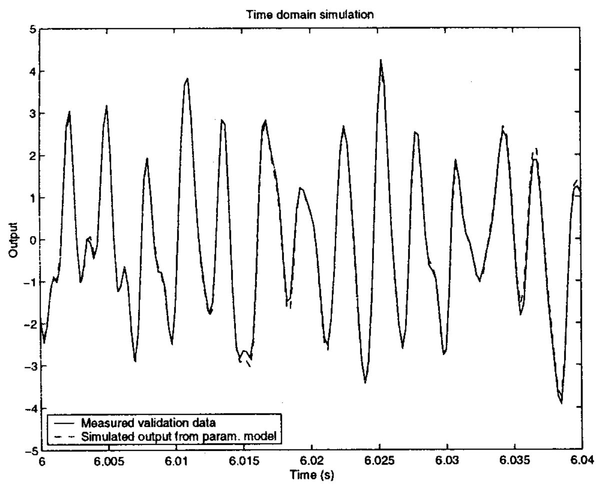

Figure 5: Time domain comparison between the simulated output of the estimated model and the measured output.

Conclusions

In this paper we successfully identified a dynamical model for an acoustic enclosure. Due to the high order of the model a particular identification procedure had to be employed. Firstly an initial model of the system was estimated using a frequency domain subspace based identification algorithm. In a second step the estimate was refined by minimizing the 2-norm of the frequency error between the model and data. This was achieved using a tridiagonal parametrization of a state-space model utilizing a Gauss-Newton type optimization algorithm.

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