DYNAMICS AND STABILITY OF WIDEBAND VIBRATION ABSORBERS WITH MULTIPLE PIEZOELECTRIC TRANSDUCERS [1]

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Abstract: This paper is concerned with the problem of dynamics and stability of piezoelectric laminate structures, in which several piezoelectric elements are shunted by a multi-input impedance. The problem is shown to be equivalent to a multivariable feedback control problem. A parameterization of stabilizing admittance transfer function matrices is given, and a specific class of controllers capable of reducing structural vibrations and guaranteeing closed loop stability is introduced. Experimental results are presented, demonstrating the effectiveness of the proposed methodology.

Keywords: Vibration Control; Piezoelectric Transducer; Passive Control.

1. INTRODUCTION

Piezoelectric transducers have been used extensively as actuators, and also as sensors in active control of structural vibrations. In such applications the piezoelectric transducer performs a single function, either as a sensor, or as actuator. However, due to the nature of the piezoelectric effect, it is possible to combine both functions in a single device.

Recently, there has been increasing interest in passive control of vibrations by shunting piezoelectric transducers with electrical impedances. This process effectively integrates sensing and actuation capabilities within a single piezoelectric transducer. An analysis of this procedure is given in (Hagood and von Flotow, 1991), where the authors suggest that a piezoelectric transducer can be shunted by a series combination of a resistor and an inductor. The piezoelectric transducer is modeled as a voltage source in series with a capacitor. The resulting RLC circuit is tuned to one of the resonance frequencies of the structure to suppress structural vibrations due to that specific mode.

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The method suggested in (Hagood and von Flotow, 1991), although effective, can only be applied to one vibration mode. However, following their work a number of authors attempted to extend this technique to allow for passive damping of several modes. In (Wu, 1998), the author proposed the use of current blocking circuits to separate RL branches tuned to each resonance frequency. The method works well for small number of modes. However, as the number of modes increases so does the complexity of the electric shunt, resulting in implementation difficulties.

Reference (Hollkamp, 1994) suggested parallel combination of a series RL circuit with several series RLC branches. The author demonstrated the effectiveness of this specific structure in reducing vibrations due to two modes of a structure experimentally. However, the synthesis procedure is not straightforward, making it difficult to extend the application to more modes.

The use of parallel combination of series RLC branches was studied in (Behrens and Moheimani, 2001). The idea is to introduce "current flowing" RC circuits in each RL branch. Complexity of the electrical shunt proposed in (Behrens and Moheimani, 2001) is considerably less than that proposed in (Wu, 1998). However, the freedom in choice of the capacitive, or alternatively the inductive elements may complicate the design process.

This paper is concerned with the problem of multi-mode shunt damping of structural vibrations using several piezoelectric transducers. To the authors' knowledge this problem has not been addressed in the literature so far. It is shown that the problem can be cast as a multivariable feedback control problem, in which the impedance, or alternatively the admittance of the electrical shunt, is the feedback controller. Dynamics of a flexible structure consists of a large number of highly resonant modes, where often only a limited number of low frequency modes are to be controlled. Conditions under which the closed loop system remains stable, in the presence of uncontrolled out-of-bandwidth modes, are derived and a number of specific structures for the electrical shunt are proposed.

2. DYNAMICS OF A SHUNTED PIEZOELECTRIC LAMINATE STRUCTURE

Consider a flexible structure with m piezoelectric patches bonded to its either side in a collocated pattern. Furthermore, assume that the piezoelectric transducers on one side are used to disturb the structure, while those on the other side of the structure are shunted to an impedance. The impedance is to be designed in a way that the unwanted structural vibrations are minimized. It should be noted that the disturbances acting on the structure can take different forms. Nevertheless, the methodology developed in this paper is general enough to apply to such cases. This point will be further clarified soon. In Figure 2.1 (a) a schematic of this system is depicted while the equivalent electrical circuit of the shunted piezoelectric transducers are drawn in Figure 2.1 (b). In this section we derive dynamics of the shunted system.

Let

$$V_z(s) = [\, v_{z_1}(s) \;\; v_{z_2}(s) \;\; \cdots \;\; v_{z_m}(s) \,]^T$$
$$V_p(s) = [\, v_{p_1}(s) \;\; v_{p_2}(s) \;\; \cdots \;\; v_{p_m}(s) \,]^T$$
$$V_{in}(s) = [\, v_{in_1}(s) \;\; v_{in_2}(s) \;\; \cdots \;\; v_{in_m}(s) \,]^T$$
$$I_z(s) = [\, i_1(s) \;\; i_2(s) \;\; \cdots \;\; i_m(s) \,]^T .$$

Then,

$$V_z(s) = Z(s) I_z(s). \tag{2.1}$$

Furthermore, writing the KVL around the k^{th} loop we obtain $v_{z_k} = v_{p_k} - \frac{1}{C_{p_k} s} i_k$ which implies

$$V_z(s) = V_p(s) - \frac{1}{s} \Lambda I_z(s) \tag{2.2}$$

where $\Lambda = \operatorname{diag}\left(\frac{1}{C_{p1}}, \frac{1}{C_{p2}}, \ldots, \frac{1}{C_{pm}} \right)$.

To capture the total effect of the disturbance voltages as well as the effect of the electric shunt on the structure, one may write (Hagood and von Flotow, 1991)

$$V_p(s) = G_{vv}(s) V_{in}(s) - G_{vv}(s) V_z(s). \tag{2.3}$$

Here, $G_{vv}(s)$ is the multivariable collocated transfer function matrix between the piezoelectric shunting voltages, assuming the shunt impedances $Z(s)$ are open circuit;

$$G_{vv}(s) = \sum_{k=1}^{M} \frac{\Psi_k}{s^2 + 2\zeta_k \omega_k s + \omega_k^2}, \tag{2.4}$$

where resonant frequencies are ordered such that $\omega_1 \leq \omega_2 \leq \ldots \leq \omega_M$ and M can be an arbitrarily large number. Furthermore, due to the fact that $G_{vv}(s)$ is a collocated transfer function matrix, we must have (Halim and Moheimani, 2001)

$$\Psi_k = \Psi_k' \geq 0 \quad \text{for all} \quad k. \tag{2.5}$$

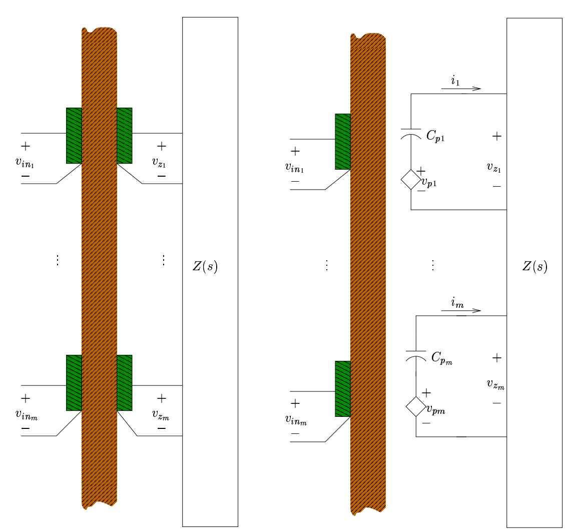
It should be pointed out that if equation (2.4) is obtained by employing a procedure such as modal analysis (Meirovitch, 1986), one would expect to have $M \longrightarrow \infty$. However, choosing a very large number for M is quite acceptable, as pointed out in (Hughes, 1987). This would allow one to use finite-dimensional techniques in analyzing dynamics of the system.

Next, equations (2.1), (2.2) and (2.3) are combined to obtain,

$$V_p(s) = \left[I + G_{vv}(s) Z(s) \left(Z(s) + \frac{1}{s}\Lambda \right)^{-1} \right]^{-1} \times G_{vv}(s) V_{in}(s). \tag{2.6}$$

From equation (2.6) it can be inferred that the transfer function matrix relating $V_{in}(s)$ to $V_p(s)$ is the feedback connection of $G_{vv}(s)$ with

$$K(s) = Z(s) \left(Z(s) + \frac{1}{s}\Lambda \right)^{-1}. \tag{2.7}$$



(a) (b)

Fig. 2.1. (a) A piezoelectric laminate structure with m shunted piezoelectric patches (b) Electrical equivalent of part (a).

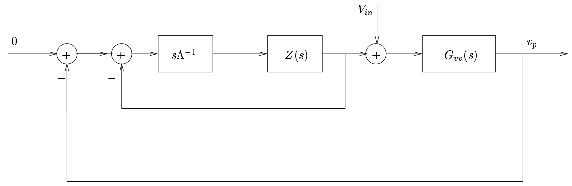


Fig. 2.2. The feedback structure associated with the shunt damping problem.

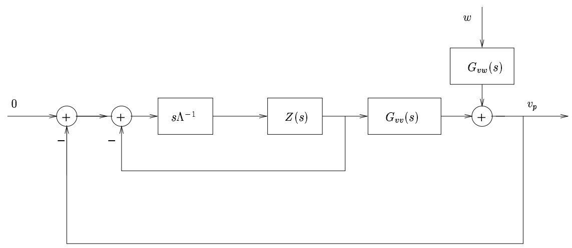


Fig. 2.3. The feedback structure associated with the modified shunt damping problem.

This is an interesting observation as it enables one to employ systems theoretic tools in analyzing dynamics and stability of shunt-damped systems. The feedback control problem associated with (2.6) is depicted in Figure 2.2. Note that the inner feedback loop represents the effective controller $K(s)$ in (2.7). Observe that the purpose of the system is to regulate v_p, in presence of a disturbance V_{in}.

The above system is mainly used in laboratory experiments. Indeed experimental results of this paper are obtained from a simply supported beam with two pairs of collocated piezoelectric transducers (see Section 5 below). In a more realistic setting, the disturbances acting on the structure have a different nature; e.g., point forces, moments, a distributed force, etc. In this situation equation (2.3) should be modified to

$$V_p(s) = G_{vv}(s)V_{in}(s) - G_{vw}(s)W(s) \qquad (2.8)$$

where $G_{vw}(s)$ is the unshunted transfer function from the disturbance vector, $W(s)$ to $V_p(s)$. An implication of equation (2.8) is that the shunted structural dynamics will have to be revised as:

$$V_p(s) = \left[I + G_{vv}(s)Z(s)\left(Z(s) + \frac{1}{s}\Lambda\right)^{-1}\right]^{-1} \times G_{vw}(s)W(s). \qquad (2.9)$$

Observe that although the nature of the disturbance has changed, stability of the shunted system is still dictated by the feedback connection of $G_{vv}(s)$ and $K(s)$ in (2.7). Furthermore, it is noted that under these circumstances the regulator problem depicted in Figure 2.2 should be modified to that shown in Figure 2.3.

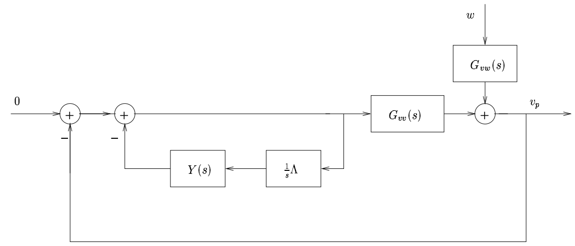


Fig. 3.1. The feedback structure associated with the shunt damping problem with admittance as the control variable.

3. STABILITY OF THE SHUNTED SYSTEM

A set of conditions under which stability of the closed loop system depicted in Figure 2.3 is guaranteed, are derived in this section. Instead of considering the shunting impedance, $Z(s)$ as the controller, the closed loop stability of the system is studied in terms of the shunted admittance, $Y(s) = Z(s)^{-1}$ noting that the closed loop transfer function in (2.9) can be re-written as:

$$V_p(s) = \left[I + G_{vv}(s)\left(I + \frac{1}{s}\Lambda Y(s)\right)^{-1}\right]^{-1} \times G_{vw}(s)\ W(s). \qquad (3.1)$$

The regulator problem associated with this system is depicted in Figure 3.1. A parameterization of stabilizing controllers for the system in (3.1) is introduced next. Considering the structure of the feedback system, the Youla parameterization of all stabilizing controllers for the inner feedback loop can be written as $Y(s) = (I - Q(s)\Lambda/s)^{-1}\, Q(s)$.

Although the inner loop contains an integrator, the parameterization for a stable plant can be used as long as $Q(s)$ satisfies a number of conditions. Namely, $Q(s)$ must be stable, proper and have a transmission zero at the origin. Furthermore, $I - Q(s)\Lambda/s$ must have a transmission zero at $s = 0$. These conditions can be enforced by choosing $Q(s) = H(s)\Lambda^{-1}s$ where $H(s)$ is stable, strictly proper and $I - H(s)$ has a zero at the origin, i.e., $I - H(s) = sJ(s)$. This choice for $Q(s)$ results in a closed loop system with the transfer function matrix

$$[I + s\ G_{vv}(s)J(s)]\, G_{vw}(s). \qquad (3.2)$$

It is now possible to find closed loop stability conditions in terms of $J(s)$ as the stability of (3.2) is equivalent to that of the system depicted in Figure 3.2.

Next, a proof is given that the closed loop system will be stable as long as $J(s)$ is a strictly positive real transfer function matrix. The following two definitions and the subsequent theorem due to (Joshi and Gupta, 1996) are needed in the proof.

Definition 3.1 An $m \times m$ rational matrix $G(s)$ is said to be positive real (PR) if

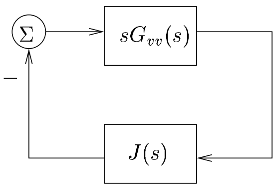


Fig. 3.2. Feedback connection of $sG_{vv}(s)$ with $J(s)$.

(i) All elements of $G(s)$ are analytic in $\mathrm{Re}(s) > 0$;
(ii) $G(s) + G^*(s) \geq 0$ in $\mathrm{Re}(s) > 0$ or equivalently
   (a) Poles on the imaginary axis are simple and have nonnegative residues, and
   (b) $G(j\omega)+G^*(j\omega) \geq 0$ for $\omega \in (-\infty, \infty)$.

Definition 3.2 An $m \times m$ stable rational matrix $G(s)$ is said to be strictly positive real in the weak sense (WSPR) if

$$G(j\omega) + G^*(j\omega) > 0 \quad \text{for} \quad \omega \in (-\infty, \infty).$$

The following theorem is Corollary 1.1 of (Joshi and Gupta, 1996).

Theorem 3.1. The negative feedback connection of a PR system with a WSPR controller is stable.

It should be pointed out that there are a number of definitions in the literature for strictly positive real systems. For a review of these, the reader is referred to (Wen, 1998; Joshi and Gupta, 1996). For almost all such definitions, one would expect a similar result to that of Theorem 3.1; i.e., the negative feedback connection of a PR system with a SPR controller is stable. It turns out that for the problem at hand, definition 3.2 is the most relevant.

First, it is proved that

$$\tilde{G}_{vv}(s) = s\ G_{vv}(s) \tag{3.3}$$

is a positive real transfer function matrix. It can be noticed from (3.3) and (2.4) that all of the poles of $\tilde{G}_{vv}(s)$ are in the left half of the complex plane, hence the system is stable. Furthermore, the system has no poles on the $j\omega$ axis. To prove positive realness of $\tilde{G}_{vv}(s)$, one needs to establish that $\tilde{G}_{vv}(j\omega) + \tilde{G}^*_{vv}(j\omega) \geq 0$ for all $\omega \in (-\infty, \infty)$, i.e. $\tilde{G}_{vv}(j\omega) + \tilde{G}^*_{vv}(j\omega) = \sum_{k=1}^{N} \frac{4\zeta_k \omega_k \omega^2 \Psi_k}{(\omega_k^2-\omega^2)^2+(2\zeta_k\omega_k\omega)^2} \geq 0$ for all $\omega \in (-\infty, \infty)$ where the last inequality follows from (2.5).

An implication of the above analysis is that to guarantee the closed loop stability of the system it would suffice to choose an admittance $Y(s) = J(s)^{-1}(I - s\ J(s))\Lambda^{-1}$ with $J(s)$ a WSPR and strictly proper transfer function matrix.

4. DECENTRALIZED CONTROL DESIGN

The observation made in the previous section enables one to design impedance structures that guarantee closed loop stability of the shunted system. This section introduces two specific decentralized structures that enforce the above conditions. Furthermore, these decentralized impedances result in effective wideband reduction of vibrations of the base structure.

These admittances are constructed starting from:

$$\hat{J}(s) = \sum_{i=1}^{N} \mathrm{diag}\left(\hat{K}_1(s), \ldots, \hat{K}_m(s)\right) \tag{4.1}$$

with $\hat{K}_l(s) = \frac{\alpha_{li}(s+2d_{li}\omega_i)}{s^2+2d_{li}\omega_i s+\omega_i^2}$ for $l = 1, 2, \ldots, m$ and

$$\tilde{J}(s) = \sum_{i=1}^{N} \mathrm{diag}\left(\tilde{K}_1(s), \ldots, \tilde{K}_m(s)\right) \tag{4.2}$$

where $\tilde{K}_l(s) = \frac{\alpha_{li}s}{s^2+2d_{li}\omega_i s+\omega_i^2}$, $l = 1, 2, \ldots, m$. Also, in both cases,

$$\alpha_{qi} \geq 0, \quad i = 1, 2 \ldots, N \quad q = 1, 2, \ldots, m. \tag{4.3}$$

and

$$\sum_{i=1}^{N} \alpha_{qi} = 1, \quad q = 1, 2, \ldots, m. \tag{4.4}$$

It can be verified that both $\hat{J}(s)$ and $\tilde{J}(s)$ are strictly proper WSPR systems. Hence, the resulting admittances will guarantee closed loop stability of the system.

Corresponding to $\hat{J}(s)$ and $\tilde{J}(s)$, the expressions for $\hat{Y}(s)$ and $\tilde{Y}(s)$ can be determined as:

$$\hat{Y}(s) = \mathrm{diag}\left(\hat{\phi}_1(s), \ldots, \hat{\phi}_m(s)\right)\Lambda^{-1}s \tag{4.5}$$

and

$$\tilde{Y}(s) = \mathrm{diag}\left(\tilde{\phi}_1(s), \ldots, \tilde{\phi}_m(s)\right)\Lambda^{-1}s. \tag{4.6}$$

where

$$\hat{\phi}_l(s) = \frac{\sum_{i=1}^{N} \frac{\alpha_{li}\omega_i^2}{s^2+2d_{li}\omega_i s+\omega_i^2}}{1 - \sum_{i=1}^{N} \frac{\alpha_{li}\omega_i^2}{s^2+2d_{li}\omega_i s+\omega_i^2}},$$

$$\tilde{\phi}_l(s) = \frac{\sum_{i=1}^{N} \frac{\alpha_{li}(2d_{li}\omega_i s+\omega_i^2)}{s^2+2d_{li}\omega_i s+\omega_i^2}}{1 - \sum_{i=1}^{N} \frac{\alpha_{li}(2d_{li}\omega_i s+\omega_i^2)}{s^2+2d_{li}\omega_i s+\omega_i^2}},$$

for all $l = 1, 2, \ldots, m$.

One of the interesting properties of the above admittance transfer functions is that in a specific bandwidth, one may choose to control only those modes that are of importance. This is reflected in the constraint on parameters α_{qi} in (4.3). This is in contrast to control design methodologies such as LQG and $\mathcal{H}_\infty$ where the controller tends to have equal dimension to that of the system that is being controlled.

A further property of the controllers $\hat{Y}(s)$ and $\tilde{Y}(s)$ is that in presence of out of bandwidth modes of the base structure they do not cause instabilities. The spill-over effect (Balas, 1978*a*; Balas, 1978*b*) is a serious cause of concern in control design for flexible structures. Often a feedback controller is designed using a model of the structure that contains a limited number of modes. Once the controller is implemented on the full order system, the presence of uncontrolled high frequency modes may destabilize the closed loop system, or severely deteriorate the performance. Considering the discussion in Section 3, it should be clear that such a problem can not happen here.

Now, it is straightforward, but tedious, to verify that both $\hat{Y}(s)$ and $\tilde{Y}(s)$ are strictly positive real transfer functions. Therefore, they can be realized by passive circuit components; i.e., resistors, inductors, and capacitors. Given that both $\hat{Y}(s)$ and $\tilde{Y}(s)$ have decentralized structures, effectively each piezoelectric transducer is shunted by an independent admittance. However, it is not clear how such a network may be obtained as standard synthesis techniques result in realizations that require Gyrators and operational amplifiers. To this end, it should be pointed out that even if passive realizations for (4.5) and (4.6) are found, in practice, such an implementation is likely to be impractical. Given that often low frequency modes of a structure are targeted for shunt damping, the required inductors may be excessively large, in the order of several hundred to several thousand Henries. A practical way of implementing $\hat{Y}(s)$ and $\tilde{Y}(s)$ is to use the synthetic admittance circuit described in (Fleming *et al.*, 2000).

5. EXPERIMENTAL RESULTS

To validate the proposed concepts, experiments were carried out at the Laboratory for Dynamics and Control of Smart Structures [2], on a piezoelectric laminated beam. Figure 5.1 shows the simply supported beam apparatus. The structure consists of a uniform aluminum beam of a rectangular cross section, refer to Table 5.1 for beam parameters, and has experimentally pinned boundary conditions at both ends. A pair of piezoelectric ceramic patches are attached symmetrically to either side of the beam, at $x_1 = 0.05m$ and $x_2 = 0.24m$, as shown in Figure 5.1. Two piezoelectric elements will be used as an actuators to generate a disturbance and the opposing elements as shunting layers. The piezoceramic elements used on the experimental composite structure are PIC151 patches. The physical parameters for these piezoelectric patch are given in Table 5.2.

During the experiment, the simply supported beam was excited using two piezoelectric actuators with a swept sine waveform from a Hewlett Packard 35670A signal analyzer. The swept sine signal was then amplified using a high voltage power amplifier capable of driving highly capacitive loads. To find the multivariable collocated transfer functions matrix $G_{vv}(s)$, the input channel of the signal analyzer was used to measure the voltage across the shunting piezoelectric layer. Using a Polytec laser scanning vibrometer (PSV-300) the unshunted multivariable transfer function matrix from the actuator voltage $V_{in}(s)$ to the displacement at a point on the beam structure, located at $r_1 = 0.170m$, $G_{rv}(s)$, as shown in Figure 5.1 was measured. The response for $G_{vv}(s)$ and $G_{rv}(s)$ were captured by the signal analyzer. Experimental results are displayed in Figures 5.2 and 5.3, for the first three modes. The reader may note that the composite system has two inputs, corresponding to actuators 1 and 2, and three outputs i.e. shunting layers 1, 2 and displacement, giving a MIMO system.

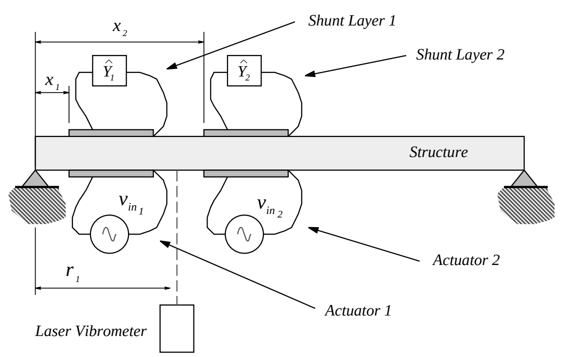


Fig. 5.1. Experimental set up of the beam apparatus.

Table 5.1. Simply supported beam parameters.

Length	$0.6m$
Width	$0.025m$
Thickness	$0.003m$
Youngs Modulus	$65 \times 10^9 N/m^2$
Density	$2650 kg/m^2$

Table 5.2. Parameters of the PIC151 piezoelectric patches.

Charge Constant	d_{31}	$-210 \times 10^{-12} m/V$
Voltage Constant	g_{31}	$-11.5 \times 10^{-3} Vm/N$
Coupling Coefficient	k_{31}	0.340
Capacitance	C_p	$0.105 \mu F$
Thickness		$0.25 \times 10^{-3} m$
Width		$0.025m$
Youngs Modulus		$63 \times 10^9 N/m^2$

In order to design an effective shunt controller a model of the multivariable system had to be obtained. Subspace based system identification techniques have proven to be an efficient means of identifying dynamics of high order, highly resonant systems (Mckelvey *et al.*, 2002). From Figures 5.2 and 5.3, it can be observed that the identified models for $G_{vv}(s)$ and $G_{rv}(s)$ describe very well the behaviour of the system.

Two different shunt circuits were applied to the beam structure, using equation (4.5), they are: $\hat{Y}_1(s)$ which controls the $2nd$ and $3rd$ modes,

[2] http://rumi.newcastle.edu.au/lab/

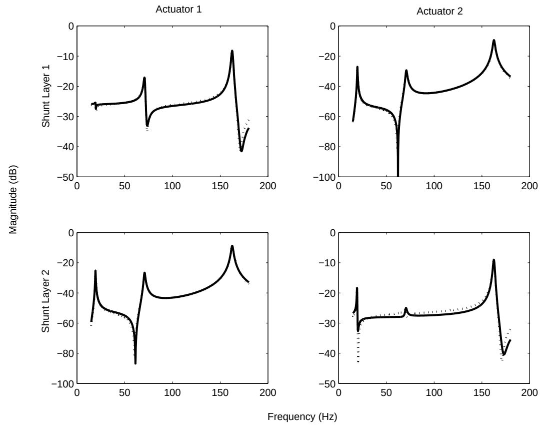


Fig. 5.2. $G_{vv}(s)$ experimental data $(\cdots)$ and identified model (—).

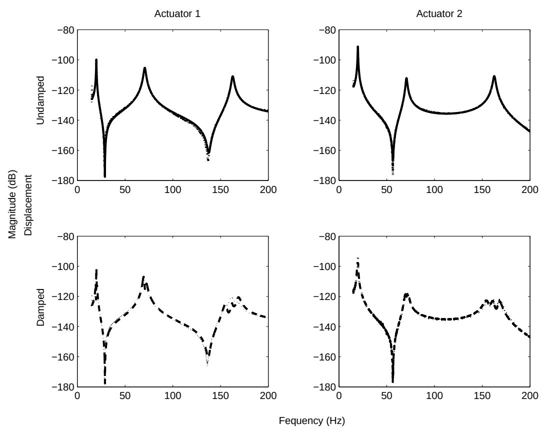


Fig. 5.3. $G_{rv}(s)$ experimental data $(\cdots)$, identified model (—) and simulated results $(-\cdot-)$.

and $\hat{Y}_2(s)$ which controls the $1st$ and $3rd$ modes. Using the predetermined parameters shown in Table 5.3 , the synthetic admittance circuit described in (Fleming *et al.*, 2000) was used to implement $\hat{Y}_1(s)$ and $\hat{Y}_2(s)$. Both admittances were shunted to the beam structure and the damped response $G_{rv}(s)$ was measured with the Polytec laser scanning vibrometer. Figure 5.3 shows the simulated and experimental damped responses. Experimental results show that the structural modes for the simply supported beam have been considerably damped; $5dB$ for the $1st$ mode, $9dB$ for the $2nd$ and $14dB$ for the $3rd$ mode.

Table 5.3. Experimental and simulated shunt parameters.

Admittance	ω_i	Hz	d_i	
$\hat{Y}_1(s)$	2	70.85	2	0.013
	3	164.3	3	0.0174
$\hat{Y}_2(s)$	1	20.54	1	0.014
	3	157.1	3	0.0173

6. CONCLUSIONS

It was demonstrated that the problem of piezoelectric shunt damping with several piezoelectric transducers and a multi-input impedance is equivalent to a feedback control problem for a square plant. The controller itself was shown to be inside an inner feedback loop. A parameterization of stabilizing controllers/electrical shunts was introduced. Two decentralized shunts with favorable damping properties were proposed and their effectiveness in reducing structural vibrations was experimentally verified.

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