THE EFFECT OF ARTIFICIALLY REDUCING THE SIZE OF INDUCTORS IN PIEZOELECTRIC SHUNT DAMPING CIRCUITS [1]

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Abstract: Connecting a passive electrical network to a structurally attached piezoelectric transducer is referred to as passive shunt damping. Current shunt circuit designs, e.g. a single mode $L-R$ network, typically require large inductance values of up to thousands of Henries. In practice, discrete inductors are limited in size to around 1 H. By placing an additional capacitance across the terminals of the piezoelectric transducer, shunt circuit inductance values can be greatly reduced. To justify our claims, we present a theoretical analysis of the damped system and identify the influence of the additional capacitance. Two modes of a simply supported beam are successfully damped using a capacitance modified shunt circuit.

Keywords: Dampers, Passive Compensation, Vibration Dampers.

1. INTRODUCTION

By shunting a structurally attached piezoelectric transducer with a passive electrical network, mechanical energy can be dissipated from the host structure. The design of shunt damping circuits has been heavily studied in the literature since the original contribution by (Hagood and A. Von Flotow, 1991), where a series inductor and resistor $R-L$ circuit was used to damp a single mode. The series inductor and resistor, when combined with the inherent capacitance of the piezoelectric transducer, creates a damped electrical resonance, equivalent to that of a tuned vibrational energy absorber (Hagood and A. Von Flotow, 1991). One highly saught after characterisic in vibration control systems is that of gauranteed stability in the presence of structural uncertainties. This quality results naturally from the fundamental properties of passive shunt damping systems (Moheimani *et al.*, 2002*a*) (Moheimani *et al.*, 2002*b*).

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Piezoelectric shunt damping circuits typically require the use of large inductances (up to thousands of Henries). Virtual grounded and floating inductors, Riodan gyrators (Riodan, 1967), are required to implement the inductor elements. Such virtual implementations are large in size, difficult to tune, and are sensitive to component non-idealities and tolerances. Virtual circuit implementations also require an external power source and large number of costly high voltage components (Fleming *et al.*, 2000*b*). Recently, a new method has been presented for implementing piezoelectric shunt damping circuits (Fleming *et al.*, 2000*a*; Fleming *et al.*, 2002). The *synthetic admittance*, comprised of a voltage controlled current source and signal filter, is capable of implementing a stable, proper admittance transfer function. The circuit contains very few high voltage components but still requires an external power supply.

This paper is aimed at reducing the inductance requirements of piezoelectric shunt damping circuits so that they may be implemented using physical components. By placing an additional capacitance

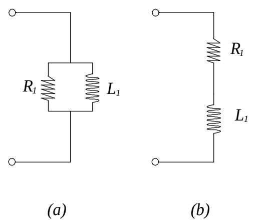

Fig. 1. Single mode shunt damping circuits.

across the terminals of the piezoelectric transducer, the effective piezoelectric capacitance can be increased. The corresponding shunt damping circuit contains reduced values of inductance. A performance penalty is involved.

We begin with a brief review of single and multimode piezoelectric shunt circuit design. In Section 3, we discuss the modelling of piezoelectric transducers and show how to model the presence of an electrical shunt impedance. The effect of an additional piezoelectric capacitance is modelled in Section 4. Experimental results are presented in Section 5. We conclude with a review of the initial goals and a summary of the theoretical and experimental results.

2. PIEZOELECTRIC SHUNT DAMPING

2.1 Single Mode Shunt Damping

Single mode damping was introduced to decrease the magnitude of one structural mode (Hagood and Crawley, 1991). Two examples of single mode damping are shown in Figure 1, parallel and series shunt damping. The combination of an $R-L$ shunt circuit combined with the intherent capacitance of the piezoelectric transducer introduces an electrical resonance. This can be tuned to one structural mode in a manner analogous to that of a mechanical vibration absorber. Single mode damping can be applied to reduce several structural modes with the use of as many piezoelectric patches and damping circuits.

Problems may result if these piezoelectric patches are bonded to, or imbedded in the structure. First, the structure may not have sufficient room to accommodate all of the patches. Second, the structure may be altered or weakened when the piezoelectric patches are applied. In addition, a large number of patches can increase the structural weight, making it unsuitable for applications such as aerospace.

2.2 Multiple Mode Shunt Damping

To alleviate the problems associated with single mode damping, multimode shunt damping has been introduced, i.e. the use of one piezoelectric patch to damp several structural modes. Two multimode shunt damping methodologies will be discussed: Current blocking techniques as presented in (Wu, 1998), and current flowing techniques as presented in (Behrens *et al.*, 2002; Behrens and Moheimani, 2002).

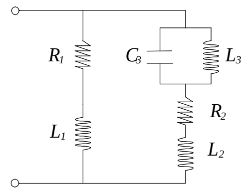

Fig. 2. A series multimode shunt damping circuit.

2.2.1. Current Blocking Techniques The principle of multimode shunt damping is to insert a *current blocking* network (Wu, 1998), in series with each shunt branch. In Figure 2, the blocking circuit consists of a capacitor and inductor in parallel, $C_3 - L_3$. As many antiresonant circuits are required in each branch as there are structural modes to be damped simultaneously. Each $R-L$ shunt branch is designed to damp one structural mode. For example, $R_1 - L_1$ in Figure 2 is tuned to resonate at ω_1, the resonance frequency of the first structural mode to be damped. $R_2 - L_2$ is tuned to ω_2, the second structural mode to be damped, and so on.

According to Wu (Wu, 1998), the inductance values for the shunt circuits shown in Figure 2 can be calculated from the following expressions, assumed that $\omega_1 < \omega_2$.

$$L_1 = \frac{1}{\omega_1^2 C_p} \;\; \tilde{L}_2 = \frac{1}{\omega_2^2 C_p} \;\; L_3 = \frac{1}{\omega_1^2 C_3} \qquad (1)$$

$$L_2 = \frac{\left(L_1\tilde{L}_2 + \tilde{L}_2 L_3 - L_1 L_3 - \omega_2^2 L_1 \tilde{L}_2 L_3 C_3\right)}{\left(L_1 - \tilde{L}_2\right)(1 - \omega_2^2 L_3 C_3)}$$

where C_p is the capacitance of the piezoelectric transducer, and C_3 is an arbitrary capacitor used in the current blocking network.

2.2.2. Current Flowing Techniques One problem with the previous technique is that the order of the shunt circuit increases quadratically as the number of modes to be shunt damped increases. Current flowing circuits (Behrens and Moheimani, 2002; Behrens *et al.*, 2002), such as that pictured in Figure 3, are easier to tune and increase in order only linearly as a greater number of modes are to be shunt damped simultaneously.

At a specific frequency ω_i, the inductor capacitor network $C_i - \widehat{L}_i$ allows current to flow through the rest of the branch, at all other frequencies the network appears approximately as an open circuit. The damping inductor and resistor $\widetilde{L}_i - R_i$ act like a single mode shunt circuit at the frequency ω_i. The circuit is simplified by combining the series inductors.

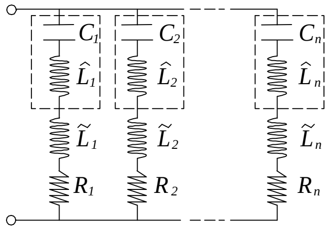

Fig. 3. Current flowing piezoelectric shunt damping circuit (Behrens and Moheimani, 2002; Behrens *et al.*, 2002).

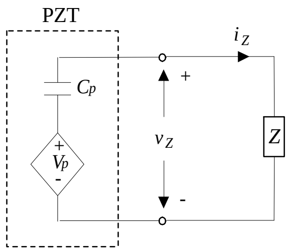

Fig. 4. Equivalent electrical model of a piezoelectric transducer.

3. MODELLING THE COMPOUND SYSTEM

For generality, we will enter the modelling process with knowledge of the system dynamics *a priori*. As an example we will consider a simply supported beam with two bonded piezoelectric patches, one to be used as a source of disturbance, and the other for shunt damping. The transfer function $G_{vv}(s)$ from an applied actuator voltage to the induced sensor voltage can be derived analytically from the Euler-Bernouli beam equation (Fuller *et al.*, 1996). Alternatively, this transfer function can be obtained experimentally through system identification (Ljung, 1999). Using similar methods, we will also consider the transfer function from applied actuator voltage to displacement at a point $G_{yv}(x, s)$.

Following the modal analysis procedure (Meirovitch, 1996), the resulting transfer functions have the familiar form,

$$G_{yv}(x,s) \triangleq \frac{Y(x,s)}{V_a(s)} = \sum_{i=1}^{\infty} \frac{F_i \phi_i(x)}{s^2 + 2\zeta_i w_i s + w_i^2}$$

$$G_{vv}(s) \triangleq \frac{V_s(s)}{V_a(s)} = \sum_{i=1}^{\infty} \frac{\alpha_i}{s^2 + 2\zeta_i w_i s + w_i^2}. \quad (2)$$

where $Y(x, s)$ is the displacement at a point, $V_s(s)$ is the piezoelectric sensor voltage, and $V_a(s)$ is the actuator voltage. F_i, and α_i represent the lumped modal and piezoelectric constants applicable to the i^{th} mode of vibration.

3.1 Piezoelectric Modelling

Piezoelectric crystals have a three-dimensional structure, i.e. crystal deformation occurs in 3 dimensions. Practical mechanical applications require the effect in one or two dimensions only, this can be achieved by manufacturing piezoelectric patches with large length and width to thickness ratios.

Piezoelectric transducers behave electrically like a capacitor and mechanically like a stiff spring (Janocha, 1999). An equivalent electrical model has been presented (Dosch *et al.*, 1992; Edberg *et al.*, 1992; Hagood and A. Von Flotow, 1991; Won, 1995) and is widely used in the literature.

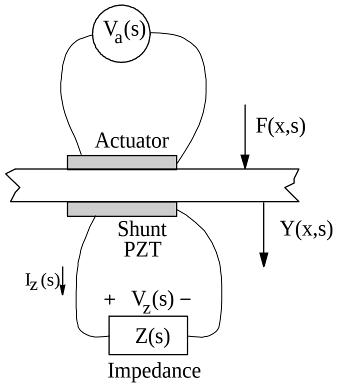

Fig. 5. Structural inputs/outputs.

The model, shown in Figure 4, consists of a strain dependent voltage source and series capacitor.

3.2 Modelling the Presence of a Shunt Circuit

Consider Figures 4 and 5 where a piezoelectric patch is shunted by an impedance Z. The current-voltage relationship can be represented in the Laplace domain as

$$V_z(s) = I_z(s)Z(s) \quad (3)$$

where $V_z(s)$ is the voltage across the impedance and $I_z(s)$ is the current flowing through the impedance. Using Kirchhoff's voltage law on the circuit shown in Figure 4, we obtain

$$V_z(s) = V_p(s) - \frac{1}{C_p s} I_z(s) \quad (4)$$

where V_p is the voltage induced by the electromechanical coupling effect (Hagood and A. Von Flotow, 1991) and C_p represents the capacitance of the shunting layer. Combining (3) and (4) we obtain

$$V_z(s) = \frac{Z(s)}{\frac{1}{C_p s} + Z(s)} V_p(s) \quad (5)$$

or

$$V_z(s) = \frac{C_p s Z(s)}{1 + C_p s Z(s)} V_p(s). \quad (6)$$

Notice that when $Z = \infty$, i.e. open-circuit, we have

$$V_z(s) \triangleq V_p(s) = G_{vv}(s)V_a(s). \quad (7)$$

However, if the circuit is shunted by Z, we can assume that

$$V_p(s) = G_{vv}(s)V_a(s) - G_{vv}(s)V_z(s). \quad (8)$$

The above equations (7) and (8) are reported in state-space form (Hagood *et al.*, 1990) as the *sensing* and *actuator equations.* By substituting (5) into (8) and rearranging we find the shunt damped transfer function

$$\frac{V_p(s)}{V_a(s)} = \frac{G_{vv}(s)}{1 + G_{vv}(s)K(s)} \quad (9)$$

where

$$K(s) = \frac{Z(s)}{Z(s) + \frac{1}{C_p s}}. \quad (10)$$

We can rewrite the shunt damped or closed loop transfer functions as

$$\widetilde{G}_{vv}(s) \triangleq \frac{V_s(s)}{V_a(s)} = \frac{G_{vv}(s)}{1 + G_{vv}(s)K(s)} \quad (11)$$

and

$$\widetilde{G}_{yv}(x,s) \triangleq \frac{Y(x,s)}{V_a(s)} = \frac{G_{yv}(x,s)}{1 + G_{vv}(s)K(s)}. \quad (12)$$

From Equations (11) and (12), we observe that shunt damping is equivalent to a negative feedback control strategy parameterized in $Z(s)$.

Using a similar procedure and the principle of superposition, the effect of a generally distributed disturbance force can be included (Moheimani *et al.*, 2002*b*).

4. SUPPLEMENTARY CAPACITANCE

The foremost problem with shunt damping implementation is the large required inductances.

Consider a shunt circuit employed to damp a single structural mode centered at 100 Hz, assuming a typical piezoelectric capacitance of 100 nF, the required inductance is 25.3 H. To implement such inductance values, designers have turned to the use of opamp based virtual inductors and more recently, impedance synthesizing devices (Fleming *et al.*, 2000*b*; Fleming *et al.*, 2002). In many circumstances, it may be undesirable to use such methods as they introduce additional complexity, reliability issues, and require external power sources.

By noting that the inductance values (1) are inversely proportional to the equivalent piezoelectric capacitance, inductances can be reduced by simply increasing the capacitance. This is achieved by selecting a piezoelectric transducer with higher dielectric constant, lesser thickness, or greater surface area. Alternatively, the piezoelectric capacitance can be increased by placing an additional capacitance C_{ad} in parallel with the piezoelectric transducer.

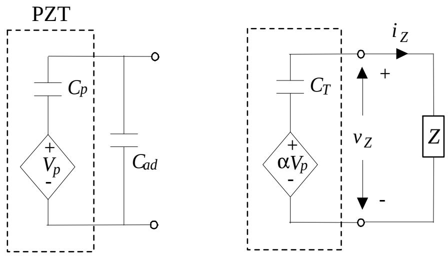

Fig. 6. Electrical and Thevenin equivalent model of the PZT with additional parallel capacitance.

The effect of the additional capacitance can be modelled by considering the Thevenin equivalent network, Figure 6. Parameters of the equivalent circuit are shown below,

$$C_T = C_p + C_{ad} \quad \alpha = \frac{C_p}{C_p + C_{ad}}. \quad (13)$$

By following a procedure similar to that of section 3.2, a model of the compound system can be obtained. The network equation (5) is replaced with

$$V_z(s) = \frac{Z(s)}{\frac{1}{C_T s} + Z(s)}\alpha V_p(s). \quad (14)$$

This is substituted into (8), yielding the closed loop transfer function from applied actuator to piezoelectric sensor voltage $\widetilde{G}_{vv}^{cap}(s)$.

$$\widetilde{G}_{vv}^{cap}(s) \triangleq \frac{V_s(s)}{V_a(s)} = \frac{G_{vv}(s)}{1 + G_{vv}(s)\alpha K(s)} \quad (15)$$

$$K(s) = \frac{Z(s)}{Z(s) + \frac{1}{C_T s}}. \quad (16)$$

Similarly,

$$\widetilde{G}_{yv}^{cap}(x,s) \triangleq \frac{Y(x,s)}{V_a(s)} = \frac{G_{yv}(x,s)}{1 + G_{vv}(s)\alpha K(s)}. \quad (17)$$

For a series configuration single mode shunt damping circuit (shown in Figure 1), the structure of the feedback controller is as follows.

$$K(s) = \frac{Z(s)}{Z(s) + \frac{1}{C_p s}}. \quad (18)$$

$$= \frac{L_1 C_p s^2 + R_1 C_p s}{L_1 C_p s^2 + R_1 C_p s + 1} \quad (19)$$

If C_p is replaced with C_T, we wish to find component values that will retain the original feedback controller $K(s)$. Simply,

Length, L	0.6 m
Width, w_b	0.05 m
Thickness, h_b	0.003 m
Youngs Modulus, E_b	65×10^9 N/m^2
Density, ρ	2650 kg/m^2

Table 1. Experimental Beam Parameters

Length	0.070 m
Charge Constant, d_{31}	-210×10^{-12} m/V
Voltage Constant, g_{31}	-11.5×10^{-3} Vm/N
Coupling Coefficient, k_{31}	0.340
Capacitance, C_p	0.105 μF
Width, w_s w_a	0.025 m
Thickness, h_s h_a	0.25×10^{-3} m
Youngs Modulus, E_s E_a	63×10^9 N/m^2

Table 2. Piezoelectric Transducer Properties

$$\widehat{L}_1 = L_1 \frac{C_p}{C_T} \quad \widehat{R}_1 = R_1 \frac{C_p}{C_T} \tag{20}$$

where $\widehat{L}_1$, and $\widehat{R}_1$ are the shunt circuit components for use with C_T. Thus, the inductance and resistance values are reduced by the same factor in which the capacitance is increased.

For multimode shunt circuits the result is similar. Because the piezoelectric capacitance has increased, branch inductance and resistance values decrease. The inductors contained in the current blocking networks are not dependent on the piezoelectric capacitance. The inductance is dependent on the arbitrary capacitance C_3, and hence is also arbitrary.

In summary, by artificially increasing the piezoelectric capacitance, inductance values can be reduced but at the expense of damping performance. Supplementary capacitance has the effect of reducing the gain of the feedback controller $K(s)$. The performance loss is a function of both the open loop dynamics and the size of the additional capacitance.

5. EXPERIMENTAL RESULTS

To validate the ideas presented, a shunt circuit with supplementary capacitance will be employed to damp two modes of a simply supported beam.

5.1 Experimental Setup

The experimental beam is a uniform aluminum bar with rectangular cross section and experimentally pinned boundary conditions at both ends. A pair of piezoelectric ceramic patches (PIC151) are attached symmetrically to either side of the beam surface. One patch is used as an actuator and the other as a shunting layer. Experimental beam and piezoelectric parameters are summarized in Tables 1 and 2.

	Natural PZT	**Modified Cap.**
C_p	104.8 nF	104.8 nF
C_{ad}	0 nF	212.2 nF
C_3	104.8 nF	317.0 nF
L_1	41.8 H	13.9 H
L_2	20.8 H	6.9 H
L_3	41.8 H	13.9 H
R_1	1543 -	514 -
R_1	1145 -	381 -

Table 3. Shunt circuit component values

	Natural PZT	**Modified Cap**
Second Mode	21.9 dB	18.0 dB
Third Mode	20.4 dB	15.7 dB

Table 4. Displacement magnitude reduction

The displacement and voltage frequency responses are measured using a Polytec laser vibrometer (PSV-300) and a HP spectrum analyzer (35670A).

For flexibility during experimentation, the shunt circuits are implemented using a synthetic impedance (Fleming *et al.*, 2000*b*; Fleming *et al.*, 2002).

5.2 Damping Performance

A traditional series configuration multimode shunt circuit is first designed using the techniques presented in (Wu, 1998; Fleming *et al.*, 2002). A summary of the component values is provided in Table 3.

The shunt circuit is then redesigned using a supplementary capacitance of 212.2 nF. A three times reduction in component values is achieved.

A summary of the damping performance is provided in Table 4. Figure 7 shows the displacement magnitude frequency responses for each of the three cases: open loop, shunt damped, and shunt damped with supplementary capacitance.

6. CONCLUSIONS

Structural vibration can be reduced by shunting an attached piezoelectric transducer with an electrical impedance. Current shunt circuit designs require impractically large inductance values of up to thousands of Henries.

The size of single, and multimode shunt circuit inductors, can be reduced by placing an additional capacitance across the terminals of the piezoelectric transducer. Branch inductance and resistance values are reduced by the same factor in which capacitance is increased. Supplementary capacitance also has the consequence of reduced controller gain and damping performance. The situation can be viewed as a trade off between desired component reduction and tolerable performance loss.

Two modes of a simply supported beam are successfully damped using a capacitance modified

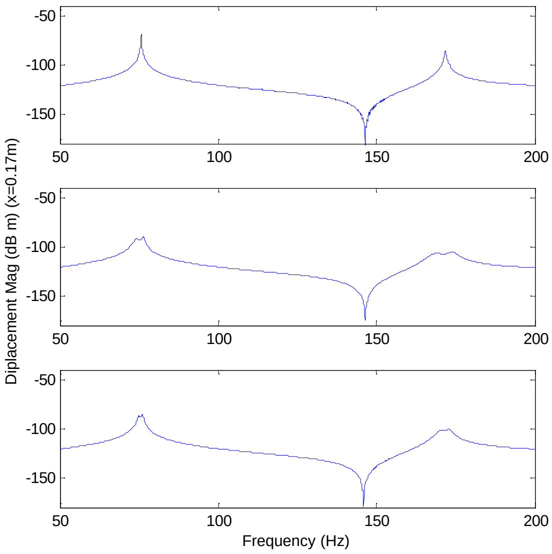

Fig. 7. Displacement magnitude frequency response, open loop, shunt damped, shunt damped with supplementary capacitance. $G_{yv}(s, x = 0.17m)$

shunt circuit. The second and third modes of a simply supported beam are reduced in magnitude by 19 and 15.7 dB. These results show a 3.9 and 4.7 dB reduction in damping performance caused by the introduction of the additional capacitance.

This concept can also be used to simplify the procedure of fine tuning. Large valued physical inductors are invariably of fixed design e.g. Bobbin core inductors. The interval between available inductance values allows only coarse tuning of the shunt circuit. By selecting a slightly smaller than necessary inductance, and including an additional variable capacitance, the circuit can be accurately tuned with negligible effect on the damping performance. Small variable capacitors are wide spread and easily obtained.

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