ON THE FEEDBACK STRUCTURE OF WIDEBAND PIEZOELECTRIC SHUNT DAMPING SYSTEMS [1]

S. O. R. Moheimani, A. J. Fleming, S. Behrens

Department of Electrical and Computer Engineering, University of Newcastle, Australia.

Abstract: This paper studies the feedback structure associated with piezoelectric shunt damping systems and introduces a new impedance structure for multi-mode piezoelectric shunt damping. The impedance is shown to be realizable using passive circuit components and digital implementation of the associated admittance transfer function is discussed.

Keywords: Piezoelectric transducers; vibration control; feedback.

1. INTRODUCTION

Piezoelectric transducers are under investigation as actuators and sensors for vibration control in flexible structures. These materials strain when exposed to a voltage and conversely produce a voltage when strained (Fuller *et al.*, 1996).

For vibration control purposes, piezoelectric transducers are bonded to the body of the base structure using strong adhesive material. These piezoelectric transducers can be used as sensors, actuators or both. One approach to vibration control, referred to as "the piezoelectric shunt-damping" is based on shunting the piezoelectric transducer by a passive electric circuit that acts as a medium for dissipating mechanical energy of the base structure. In their original work (Hagood and von Flotow, 1991; Hagood *et al.*, 1990) Hagood and von Flotow suggested that a series R-L circuit attached across the conducting surfaces of a piezoelectric transducer can be tuned to dissipate mechanical energy of the base structure. They demonstrated the effectiveness of this technique by tuning the resulting R-L-C circuit to a specific resonant frequency of the base structure. Furthermore, they proposed a method to determine an effective value for the resistive element.

In reference (Wu, 1998*a*) it was demonstrated that the concept can be extended to allow for multiple-mode shunt damping by introducing current blocking circuits inside each R-L branch (Wu, 1998*a*; Wu, 1998*b*). The problem with this technique, however, is that the size of the shunting circuit increases rapidly as the number of modes that are to be shunt dampened is increased. An alternative multi-mode shunt damping circuit was suggested by Hollkamp (Hollkamp, 1994). Although, the author conjectures the effectiveness of this circuit, no straightforward method for determining the circuit components is proposed.

A difficulty that often arises in implementing these shunt impedances is the fact that one may need to have access to rather large inductors if the low frequency modes are to be shunt dampened. The synthetic impedance circuit suggested in (Fleming *et al.*, 2000) is an effective means of digital implementation of an impedance circuit for piezoelectric shunt damping. This circuit allows us to implement any admittance transfer function, as long as it is stable, and at least proper.

In this paper we study the feedback structure of piezoelectric shunt damping systems. Furthermore, we propose a new class of impedances that can be used for this purpose.

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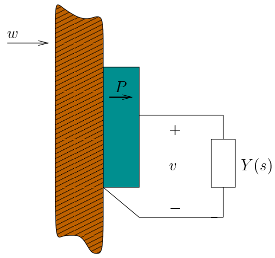

Fig. 2.1. A piezoelectric laminate shunted to an admittance $Y(s)$

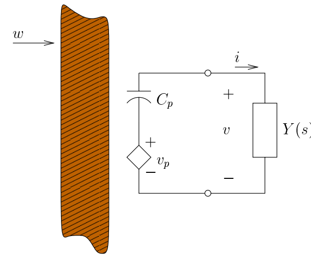

Fig. 2.2. Electrical equivalent of the system in Figure 2.1

2. FEEDBACK PROBLEM ASSOCIATED WITH A SHUNTED PIEZOELECTRIC LAMINATE STRUCTURE

Let us consider the system depicted in Figure 2.1. Here a piezoelectric transducer is attached to the surface of a flexible structure using strong adhesive material. The piezoelectric transducer is shunted to an electrical admittance, Y. The vector P signifies the direction of polarization vector of the piezoelectric material. As the structure deforms, possibly due to a disturbance w, an electric charge distribution appears inside the piezoelectric crystal. This manifests itself in the form of a voltage difference across the conducting surfaces of the piezoelectric transducer, v which in turn causes the flow of electric current, i through the admittance. This may cause loss of energy. Hence, the electric admittance may be thought of as a means of extracting mechanical energy from the base structure via the piezoelectric transducer.

To make the discussion clearer, let us look at the system in more detail. Figure 2.2 depicts the electrical equivalent of the piezoelectric transducer. If the admittance is removed from the circuit, i.e., if the piezoelectric transducer is left open circuited, then the voltage measured across the conducting terminals of the piezoelectric transducer is equivalent to v_p. This voltage is entirely due to the disturbances acting on the structure and/or non-zero initial conditions. It should be clear that as long as the base structure is not at rest, v_p may be non-zero. To this end, let us assume that v_p is related to w via a transfer function G_{vw}. That is,

$$v_p(s) = G_{vw}(s)w(s), \qquad Y(s) = 0. \quad (2.1)$$

The condition $Y(s) = 0$ in (2.1) emphasizes that this equation is valid only if the two terminals of the piezoelectric transducer are left open circuited.

Now, let us assume that there are no disturbances acting on the structure. Rather, allow us to assume that a voltage source is attached across the conducting terminals of the piezoelectric transducer. In this case, the voltage v_p is entirely due to v and is related to v via a transfer function G_{vv}. That is,

$$v_p(s) = G_{vv}(s)v(s), \qquad w(s) = 0. \quad (2.2)$$

The transfer function G_{vv} may be written in the general form (Halim and Moheimani, 2001*b*)

$$G_{vv}(s) = -\sum_{i=1}^{\infty} \frac{\gamma_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \quad (2.3)$$

where $\gamma_i > 0 \quad for \quad i = 1, 2, \ldots$.

Note that if the piezoelectric transducer is attached to the structure such that vector P is pointing to the opposite direction, the negative sign in (2.3) should be removed. If the base structure is disturbed by w and a voltage v is simultaneously applied across the terminals of the piezoelectric transducer then due to the linearity of the system we may write

$$v_p(s) = G_{vw}(s)w(s) + G_{vv}(s)v(s). \quad (2.4)$$

From equation (2.4) it can be deduced that while the disturbance w is disturbing the base structure, the voltage $v(s)$ applied across the piezoelectric terminals may be used to reduce the effect of this unwanted disturbance. In a typical feedback control problem, a sensor is used to measure a property of the structure for feedback. This may be the acceleration at some point, as measured by an accelerometer, or even the voltage measured at the open terminals of another piezoelectric transducer attached to the structure at a different point.

Shunting the piezoelectric transducer with the admittance Y, as in Figure 2.2 removes the need for an additional sensor. This, however, is achieved at the expense of having to deal with a more complicated feedback control problem.

To visualize the underlying feedback control structure, we need to identify a number of variables such as the control signal, the measurement, the disturbance and the physical variable that is to be regulated.

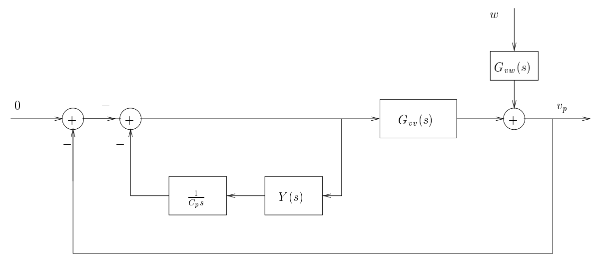

Fig. 2.3. The feedback structure associated with the shunt damping problem in Figure 2.2

The underlying feedback structure can be identified by noticing that the current may be written as:

$$i(s) = (v_p(s) - v(s))\, C_p s. \tag{2.5}$$

Furthermore,

$$i(s) = Y(s)v(s). \tag{2.6}$$

Equations (2.4)-(2.6) suggest the feedback structure depicted in Figure 2.3. The Figure suggests a rather complicated feedback structure as the controller, $Y(s)$ is itself inside an inner feedback loop.

3. A NEW IMPEDANCE STRUCTURE FOR PIEZOELECTRIC SHUNT DAMPING

In the previous section we demonstrated that the piezoelectric shunt damping problem is equivalent to a feedback control problem with a very specific feedback structure. This understanding of the underlying feedback structure allows us to interpret the existing results in the literature in a meaningful way. Furthermore, it enables us to make new contributions to the field in the form of generating new classes of high-performance shunt damping impedance structures.

Notice that in Figure 2.2 the closed loop transfer function from the disturbance input w to v_p can be written as

$$\frac{v_p(s)}{w(s)} = \frac{G_{vw}(s)}{1 + K(s)\tilde{G}_{vv}(s)} \tag{3.1}$$

where $\tilde{G}_{vv}(s) = -G_{vv}(s)$ and

$$K(s) = \frac{1}{1 + \frac{Y(s)}{C_p s}}. \tag{3.2}$$

Given the common-pole property of the transfer functions associated with the base structure, regardless of the nature of the disturbance, G_{vw} must have poles that are identical to those of $\tilde{G}_{vv}(s)$. Therefore, the role of the shunting admittance $Y(s)$ is to move the closed loop poles of the system deeper into the left half plane, i.e. to add more damping to each mode.

An effective admittance structure for this purpose is:

$$Y(s) = \frac{\sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2}}{1 - \sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2}} \cdot C_p s \tag{3.3}$$

where $\alpha_i > 0,\ d_i > 0, \quad i = 1, 2, \ldots, N$

and

$$\sum_{i=1}^{N} \alpha_i = 1. \tag{3.4}$$

An immediate choice for α's is $\alpha_i = \frac{1}{N}$ for $i = 1, 2, \ldots, N$. This will ensure that condition (3.4) is satisfied. It is straightforward to verify that for the admittance structure defined in (3.3), the effective controller expression in (3.2) will be

$$K(s) = 1 - \sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2}. \tag{3.5}$$

This, in turn can be shown to be equivalent to

$$K(s) = \sum_{i=1}^{N} \frac{\alpha_i s(s + 2d_i\omega_i)}{s^2 + 2d_i\omega_i s + \omega_i^2}. \tag{3.6}$$

It should be possible to imagine why this specific structure may be quite effective in reducing unwanted vibrations of the base structure. Flexible structures are inherently highly resonant systems whose dynamics consist of a large number of very lightly damped modes. The admittance suggested in (3.3), once shunted to the piezoelectric transducer with the piezoelectric capacitance of C_p will result in an equivalent feedback control problem where the controller $K(s)$ is defined as in (3.6). It can be observed that this controller has a highly resonant structure dictated by the damping factors $d_1, \ldots, d_N$. The controller applies a high gain at each specific resonant frequency. This is done by applying a very narrow bandpass filter around each resonant frequency of the base structure.

To see the connections with the earlier work, we point out that if $N = 1$, then the controller may be tuned only to one specific resonant frequency, say ω_ℓ. In this case, it can be shown that $Y(s) = \frac{\omega_\ell^2 C_p}{s + 2d_\ell\omega_\ell}$.

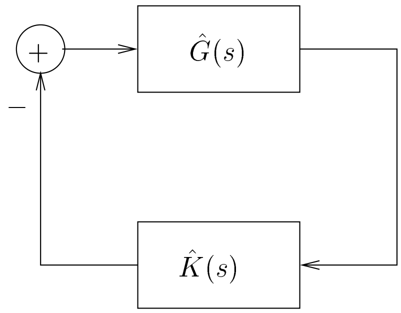

Fig. 4.1. Equivalent system for study of closed loop stability.

Hence, $Y(s)$ effectively represents the series connection of a resistor $R = \frac{2d_\ell}{\omega_\ell C_p}$ with an inductor $L = \frac{1}{\omega_\ell^2 C_p}$ shunted across the piezoelectric transducer terminals. This is the original single-mode shunt damping circuit proposed by Hagood and von Flotow (Hagood and von Flotow, 1991). Based on this observation, one may argue that $Y(s)$ in (3.3) effectively generates phase and gain relationship around each resonant frequency that is similar to those generated by a R-L circuit tuned to that specific resonant frequency. In Section 5 we will demonstrate the connections with the Hollkamp circuit (Hollkamp, 1994).

4. CLOSED LOOP STABILITY

In this section we study stability properties of the proposed shunting impedance.

By inspection, it can be verified that the closed loop stability of the system in Figure 2.3 is equivalent to the stability of the feedback loop in Figure 4.1 with

$$\begin{aligned}\hat{G}(s) &= -sG_{vv}(s)\\ &= \sum_{i=1}^{M} \frac{\gamma_i s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}\end{aligned} \quad (4.1)$$

and $K(s) = \sum_{i=1}^{N} \frac{\alpha_i s + 2d_i\omega_i}{s^2+2d_i\omega_i s+\omega_i^2}$.

The proof of closed loop stability is rather straightforward and is based on the observation that $\hat{K}(s)$ is a strictly positive real transfer function, i.e., $\hat{K}$ is stable and $\hat{K}(j\omega) + \hat{K}(-j\omega) > 0$ for all $\omega \in \mathbf{R}$ and $\hat{G}(s)$ is a positive real transfer function, i.e., $\hat{G}$ is stable and $\hat{G}(j\omega) + \hat{G}(-j\omega) \geq 0$ for all $\omega \in \mathbf{R}$. The feedback connection of two SISO systems where one is a SPR and the other is a PR transfer function is stable with a guaranteed gain margin of infinity (see Chapter 10 of (Khalil, 1996)). Therefore, the admittance suggested in (3.3) results in a closed loop system that is stable with favorable stability margins.

It should be pointed out that (4.1) with M arbitrarily large, i.e., $M \gg N$ is a reasonable finite-dimensional approximation of (2.3) (see (Hughes, 1987)).

5. PROPERTIES OF THE PROPOSED ADMITTANCE AND IMPLEMENTATION ISSUES

Our ultimate goal is to implement the admittance $Y(s)$ digitally using the synthetic admittance circuit proposed in (Fleming *et al.*, 2000). For this to be achievable in an efficient way, $Y(s)$ must satisfy a number of conditions. It should be a stable transfer function and it should be at least proper, and preferably strictly proper with a bandwidth that is not excessively larger than that of the highest in-bandwidth mode of the base structure that is to be controlled. In this section we study the structure of the proposed admittance and will show that it satisfies all the above conditions.

We first study stability of $Y(s)$. This can be verified by observing that the Nyquist plot of

$$-\sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2} \quad (5.1)$$

will never cross the critical point, $-1 + j0$. This along with the feedback structure of $Y(s)$ in (3.3) establishes the stability of the admittance $Y(s)$.

Next, we note that the admittance $Y(s)$ can be written as

$$Y(s) = \frac{\sum_{i=1}^{N} \frac{C_p\alpha_i\omega_i^2 s}{s^2+2d_i\omega_i s+\omega_i^2}}{\sum_{i=1}^{N} \frac{s(s+2d_i\omega_i)}{s^2+2d_i\omega_i s+\omega_i^2}} = \frac{H(s)}{J(s)}.$$

Now it can be verified that the numerator transfer function, $H(s)$ is a positive real transfer function, which means $-\frac{\pi}{2} \leq \angle H(s) \leq \frac{\pi}{2}$. Furthermore, it can be verified that $0 < \angle J(s) < \pi$. Hence, we may conclude that $-\frac{\pi}{2} < \angle Y(s) < \frac{\pi}{2}$, which means that $Y(s)$ is a strictly positive real transfer function, i.e., Nyquist plot of $Y(s)$ is confined to the right half of the complex plane. An implication of this observation is that $Y(s)$ is indeed realizable using purely passive circuit components, i.e., resistors, inductors and capacitors. Such a circuit may be realized by observing that $Y(s)$ can be written as

$$Y(s) = \frac{C_p \sum_{i=1}^{N} \alpha_i\omega_i^2 \prod_{\ell=1,\ell\neq i}^{N}(s^2 + 2d_\ell\omega_\ell s + \omega_\ell^2)}{\sum_{i=1}^{N}(s + 2d_i\omega_i) \prod_{\ell=1,\ell\neq i}^{N}(s^2 + 2d_\ell\omega_\ell s + \omega_\ell^2)} \quad (5.2)$$

and by employing a standard passive circuit synthesis technique. In particular, from partial frac-

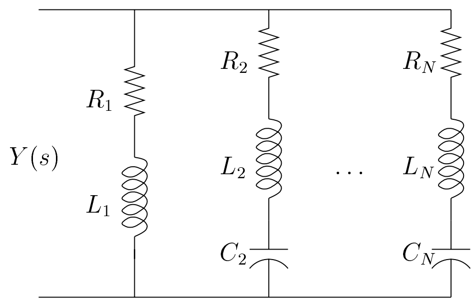

Fig. 5.1. A possible realization of (5.2).

tion expansion of (5.2) we can find the realization in Figure 5.1. This is identical to the shunt damping circuit proposed by Hollkamp (Hollkamp, 1994).

6. ROBUSTNESS ISSUES

An interesting property of the admittance proposed in (3.3) is its good robustness properties. To make this clearer we point out that under (3.3) the closed loop system is stable with a gain margin of infinity. Therefore, the spill-over effect due to the existence of out-of-bandwidth modes will not destabilize the closed loop system. As a matter of fact, the spill-over effect will be minimal since the admittance, and hence the resulting equivalent controller $K(s)$ in (3.2) has a highly resonant nature.

The structure of the admittance $Y(s)$ is such that if the resonant frequencies $\omega_1, \ldots, \omega_N$ are slightly different from the actual resonant frequencies of the base structure, closed loop stability is guaranteed. This is a favorable property as these resonant frequencies are known to change with temperature, changing load, etc.

A particularly important robustness feature of the proposed admittance structure is that it maintains closed loop stability even if the value of the piezoelectric capacitance in (3.3) is estimated incorrectly. A proof of this claim follows.

Let us assume that the actual value of the piezoelectric capacitance is C_p, while our estimate of it is $\bar{C}_p = \eta C_p$. Therefore, the admittance in (3.3) should be modified to

$$Y(s) = \frac{\sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2}}{1 - \sum_{i=1}^{N} \frac{\alpha_i \omega_i^2}{s^2 + 2d_i\omega_i s + \omega_i^2}} \cdot \bar{C}_p s.$$

Arguing along similar lines to the Section 4, we may say that the stability of the resulting closed loop system is equivalent to the stability of the system in Figure 4.1 with $\hat{G}(s) = -sG_{vv}(s)$, and $\hat{K}(s) = \frac{\frac{1}{s}}{1+\frac{\eta}{s}\tilde{Y}(s)}$, with $\tilde{Y}(s) = \frac{1}{C_p}Y(s)$.

We have already established that $Y(s)$ is a strictly positive real transfer function. Therefore strict positive realness of $\tilde{Y}(s)$ follows immediately. Now, it can be proved that $\hat{K}(s)$ is stable and that $\hat{K}(j\omega) + \hat{K}(-j\omega) > 0$ for all $\omega \in \mathbf{R}$. Therefore, $\hat{K}(s)$ is itself a strictly positive real system. Given that $\hat{G}(s)$ is a positive real system, we may conclude that the closed loop system is table for any $\eta > 0$.

To this end we point out that although the closed loop system will not be destabilized, the performance of the system may severely deteriorate as η deviates from one.

7. OPTIMAL TUNING OF THE ADMITTANCE

Structure of the admittance in (3.3) guarantees closed loop stability of the system. In order to achieve good performance, appropriate values for the damping parameters $d_1, d_2, \ldots, d_N$ need to be determined. This may be done by seeking a solution to the following optimization problem.

$$d_1^*, d_2^*, \ldots, d_N^* = \arg\min \|T_{v_p w}\|_2. \qquad (7.1)$$

This is a non-convex optimization problem that could have many local minimas. Typically, one would attempt to solve the problem using a gradient descent technique (Luenberger, 1969). In doing so, one would need to choose a starting point from which the optimization process may start. Given that for all positive $d_1, d_2, \ldots, d_N$ the closed loop system is stable, any positive value may be considered acceptable. However, considering the structure of the system, it may be possible to find a set of damping ratios reasonably close to a minima.

The transfer function $G_{vv}(s)$ in (2.3) is a high order system of very lightly damped resonant modes. Depending on the geometry of the structure, these modes may be reasonably far away from one another. Given the highly localized nature of $Y(s)$, it may be a reasonable assumption to consider the effect of each individual bandpass section of the admittance on the specific mode of the base structure. Doing so, one may then search for a value of the damping ratio that would place the closed loop poles of the system as deep into the left half of the complex plane as possible. A repeat of this procedure for every single mode that is to be controlled may result in a good initial condition for the optimization problem (7.1).

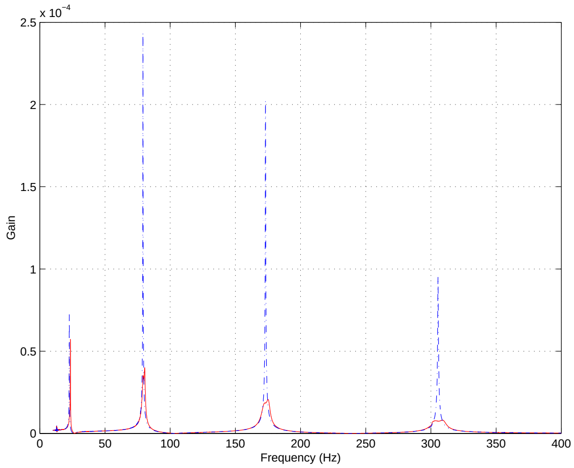

Fig. 8.1. Open loop vs. closed loop response (beam).

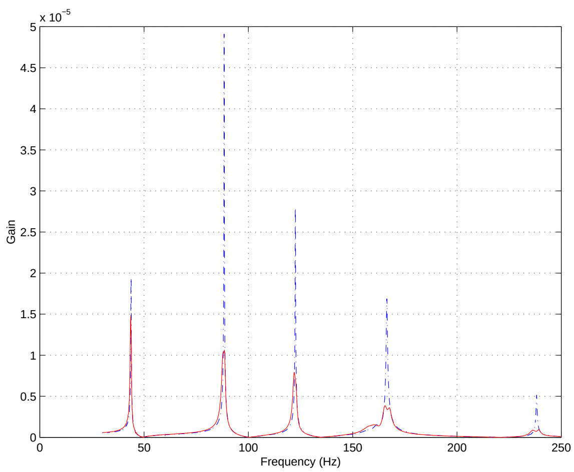

Fig. 8.2. Open loop vs. closed loop response (plate).

8. EXPERIMENTAL RESULTS

In this section we apply the above procedure to two flexible structures. These are the piezoelectric laminate beam described in (Behrens and Moheimani, 2000) and the piezoelectric laminate plate described in (Halim and Moheimani, 2001*a*). The first four modes of the beam and the first six modes of the plate are to be controlled by a shunt impedance Y with the structure given in (3.3).

A model of the composite system, $G - vv(s)$ was obtained using the frequency domain system identification (McKelvey *et al.*, 1996). The data was taken using a Hewlett Packard model 89410 Vector Analyzer. The procedure explained in the previous section was used to determine the optimal set of damping ratios for $Y(s)$ for the plate and the beam. The impedances were digitally implemented using the synthetic admittance circuit described in (Fleming *et al.*, 2000). A comparison of the closed open loop and closed loop results are shown in Figures 8.1 and 8.2. These plots are associated with the displacement transfer functions at specific points over the surfaces of the structures.

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