POWER HARVESTING PIEZOELECTRIC SHUNT DAMPING [1]

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Abstract: Passive shunt damping involves the connection of an electrical shunt network to a structurally attached piezoelectric transducer. Typical shunt networks require inductance values of up to thousands of Henries. Virtual inductors or synthetic networks are required to implement the required impedance. This paper introduces an efficient, light weight, and small-in-size technique for implementing piezoelectric shunt damping circuits. A MOSFET half bridge is used together with a signal processor to synthesize the terminal impedance of a piezoelectric shunt damping circuit. Under certain circumstances, the device is capable of harvesting the real power normally dissipated by resistive network elements.

Keywords: Dampers, Passive Compensation, Vibration Dampers, AC Converter Machines.

1. INTRODUCTION

Passive shunt damping involves the connection of an electrical shunt network to a structurally attached piezoelectric transducer. By means of the piezoelectric mechanical to electrical coupling, the passive network is capable of damping structural vibrations. If a simple resistor is placed across the terminals of the PZT, the PZT will act as a viscoelastic damper (Hagood and A. Von Flotow, 1991). If the network consists of a series inductor and resistor, the passive network combined with the inherent capacitance of the PZT creates a damped electrical resonance. The resonance can be tuned so that the PZT acts as a tuned vibrational energy absorber (Hagood and A. Von Flotow, 1991). Passive shunt damping has the property of guaranteed stability in the presence of structural uncertainty (Moheimani *et al.*, 2002).

Typical shunt damping circuits require large inductance values of up to thousands of Henries. Previously, shunt circuits have been implemented using discrete resistors, virtual inductors, and Riodan Gyrators (Riodan, 1967). The difficulty in constructing shunt circuits and achieving reasonable performance has been an ongoing problem in shunt damping. Recently a new approach has been presented for implementation of piezoelectric shunt damping circuits. The *synthetic admittance* (Fleming *et al.*, 2000; Fleming *et al.*, 2002), consisting of a current source and signal filter, is capable of maintaining an arbitrary relationship between current and voltage at its terminals. Both methods require an external power source and contain inherently inefficient linear power components. Intuitively, one may expect that passive shunt damping circuits, being dissipative, can be implemented without an external power source.

This paper first introduces a new method for implementing an arbitrary terminal admittance. The switched mode synthetic admittance requires no high voltage linear components, is small in size, and is ideal for implementing industrial scale shunt damping systems with large excitation. It is shown how the device is capable of harvesting

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power from structural vibrations. Energy that would normally be dissipated by resistive circuit components can be stored for external use.

1.1 Piezoelectric Shunt Damping

Single mode damping was introduced to decrease the magnitude of one structural mode (Hagood and Crawley, 1991). Two examples of single mode damping are shown in Figure 1, parallel and series shunt damping. An $R-L$ shunt circuit introduces an electrical resonance, this can be tuned to one structural mode in a manner analogous to that of a mechanical vibration absorber. Single mode damping can be applied to reduce several structural modes with the use of as many piezoelectric patches and damping circuits.

Problems may result if the piezoelectric patches are bonded to, or imbedded in the structure. First, the structure may not have sufficient room to accommodate all of the patches. Second, the structure may be altered or weakened when the piezoelectric patches are applied. In addition, a large number of patches can increase the structural weight, making it unsuitable for applications such as aerospace.

To alleviate the problems associated with single mode damping, multi-mode shunt damping has been introduced, i.e. the use of one piezoelectric patch to damp multiple structural modes. Several techniques have emerged:

- Current blocking techniques, as presented in (Wu, 1998), are based on placing a number of single mode $R-L$ branches in parallel, one for each mode to be controlled. Each branch also requires the addition of a parallel $L-C$ current blocking network to isolate the effect of each branch to a single mode.
- Current flowing techniques, as presented in (Behrens and Moheimani, 2002), are similar to current blocking techniques, a number of single mode branches are connected in parallel. Current flowing networks are used in place of the current blocking networks to isolate adjacent branches. Current flowing shunt damping circuits are of considerably less order than current blocking circuits and hence, prove useful for damping a large number of modes
- By considering the underlying feedback structure associated with piezoelectric shunt damping (Moheimani *et al.*, 2002), a suitable controller is first designed, then used to identify the corresponding shunt admittance.

Typical shunt circuits require large inductance values of up to thousands of Henries. Virtual grounded and floating inductors (Riodan gyrators (Riodan, 1967)) are required to implement the inductor elements. Such virtual implementations are large in size, difficult to tune, and are sensitive to component age, temperature, and non-ideal characteristics.

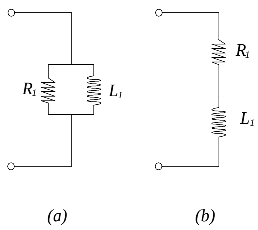

Fig. 1. Parallel (a) and series (b) single mode shunt damping circuits.

Piezoelectric patches are capable of generating hundreds of volts for moderate structural excitations. This requires that the entire circuit be constructed from high voltage components. Further voltage limitations arise due to the internal gains of the virtual inductors.

1.2 Modelling the Compound System

For generality, we will enter the modelling process with knowledge *a priori* of the system dynamics. As an example we will consider a simply supported beam with two bonded piezoelectric patches, one to be used as a source of disturbance, and the other for shunt damping. The transfer function $G_{vv}(s)$ from applied actuator voltage to sensor voltage can be derived analytically from the Euler-Bernoulli beam equation (Fuller *et al.*, 1996), or obtained experimentally through system identification (Ljung, 1999; Mckelvy *et al.*, 1996). Using similar methods, we may obtain the transfer function from an applied actuator voltage to the resulting displacement at a point $G_{yv}(x, s)$. Consider Figure 2 where a piezoelectric patch is shunted by an impedance $Z(s)$. In reference (Moheimani *et al.*, 2002), the damped system transfer functions $\widetilde{G}_{vv}(s)$, and $\widetilde{G}_{yv}(x, s)$ are shown to be,

$$\widetilde{G}_{vv}(s) \triangleq \frac{V_s(s)}{V_a(s)} = \frac{G_{vv}(s)}{1 + G_{vv}(s)K(s)}. \quad (1)$$

$$\widetilde{G}_{yv}(x, s) \triangleq \frac{Y(x, s)}{V_a(s)} = \frac{G_{yv}(x, s)}{1 + G_{vv}(s)K(s)}. \quad (2)$$

where $K(s) = \frac{Z(s)}{Z(s)+\frac{1}{C_p s}}$. Note that $V_p(s)$ is dynamically equivalent to $V_s(s)$ (i.e. the open circuit voltage). Using a similar procedure and the principle of superposition, the effect of a generally distributed disturbance force can be included (Moheimani *et al.*, 2002).

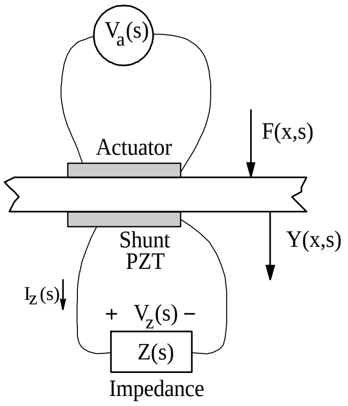

Fig. 2. Structural inputs / outputs.

2. THE SWITCHED MODE SYNTHETIC ADMITTANCE

The switched mode synthetic admittance will be introduced as an alternative to the synthetic admittance (Fleming *et al.*, 2000; Fleming *et al.*, 2002).

2.1 Device Operation

A simplified circuit diagram of the switched mode admittance is shown in Figure 3. The basic idea is the same as that discussed previously, the device attempts to maintain some arbitrary relationship between voltage and current at its terminals, i.e.,between i_T and v_T.

We begin with some preliminary circuit analysis. In the Laplace domain,

$$I_T(s) = \frac{V_T(s) - V_{pwm}(s)}{Z_c(s)}. \tag{3}$$

We desire the terminal voltages and currents to be related by some arbitrary function, in this case a terminal impedance Z_T.

$$I_T(s) = \frac{1}{Z_T(s)} V_T(s) \tag{4}$$

Combining (3) and (4) yields the relationship required to maintain (4) at the terminals,

$$V_{pwm}(s) = V_T(s)\left(1 - \frac{Z_c(s)}{Z_T(s)}\right). \tag{5}$$

The reader may recognize the similarity between the circuit on the right hand side of Figure 3 and a controlled single phase switch mode rectifier, or a four quadrant switched mode amplifier. Indeed, the only difference between such devices is the selection of the control impedance and the bridge control algorithm. Although we cannot synthesize $v_{pwm}(t)$ exactly, we can do so in the average sense. The relationship between the reference signal and the control duty cycle is $D = \frac{1}{2}\left(\frac{v_{ref}}{V_{dc}} + 1\right)$.

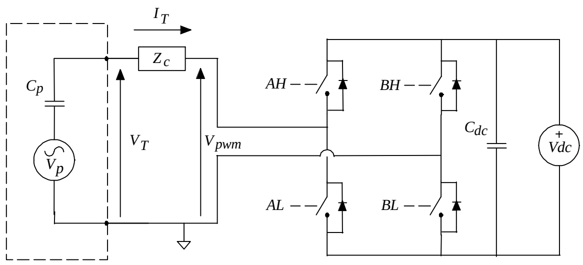

Fig. 3. The switched mode synthetic admittance.

The principle of operation is explained fairly simply. The desired terminal current (a function of the terminal voltage) is synthesized by controlling the average voltage across the impedance Z_c.

2.1.1. Boost Configuration In this section we consider a specific choice for the control impedance Z_c, a series connection of an inductor and resistor. In this configuration, the structure of the circuit resembles that of a single phase boost rectifier. The primary motivation is to allow the flow of real and reactive power back to the source.

Assuming that the inductance is large enough to maintain an approximately constant current over the switching interval, when the applied potential v_{pwm} opposes the current i_T, the inductor over comes the source potential and forces the current to flow through the anti-parallel diodes back to the source.

This configuration also has the advantage of greatly reducing the high frequency content applied to the piezoelectric transducer. The inherent capacitance of the PZT together with the control impedance creates a second order resonant low pass power filter.

$$V_T(s) = \frac{\frac{1}{L_c C_p}}{s^2 + \frac{R_c}{L_c}s + \frac{1}{L_c C_p}} V_{pwm}(s) \tag{6}$$

For reasonable values of R_c, L_c, and C_p (300 - , 0.1 H, 400 ηF), the filter has a cutoff frequency of around 800 Hz. If we consider a system with a switching frequency of 8 kHz, such a filter would attenuate the fundamental switching component by 40 dB. Taking into account the additional low pass dynamics of the plant, the actual realized disturbance due to switching is negligible.

2.2 Efficiency

If we consider a sinusoidal voltage source V_s connected to an impedance Z_T, the real dissipated power is

$$P_T = \frac{1}{2}|V_s|^2 \operatorname{Re}\left\{\frac{1}{Z_T}\right\} = \frac{1}{2}\left|\frac{V_s}{Z_T}\right|^2 \operatorname{Re}\{Z_T\} \tag{7}$$

We define the efficiency of the switch mode synthetic admittance as the ratio of power absorbed by V_{dc}, to the power that would normally be dissipated if the impedance Z_T was implemented using ideal physical components, $\eta(Z_c, Z_T, \omega) = 100\% \times \frac{P_{V_{dc}}}{P_T}$. Virtual or linear synthetic implementations will always result in a negative efficiency, i.e., they absorb no real power. In fact, the situation is worse, such implementations must actually supply power to synthesize the flow of apparent power. For our application, i.e., synthesizing inductors to form a highly resonant circuit, the realized efficiency is extremely poor (large and negative).

The quantity $P_{V_{dc}}$ is computed easily for the boost configuration by performing a power balance. Obviously, the real power as seen from the terminals will be equal to P_T. The only remaining contribution to the net real power flow is the control impedance,

$$P_c = \frac{1}{2}\left|\frac{V_s}{Z_T}\right|^2 \operatorname{Re}\{Z_c\} \tag{8}$$

$$\eta(Z_c, Z_T, j\omega) = 100\% \times \frac{P_T - P_c}{P_T} \tag{9}$$

$$= 100\% \times \left[1 - \frac{\operatorname{Re}\{Z_c\}}{\operatorname{Re}\{Z_T\}}\right]. \tag{10}$$

The best efficiency (100%) is achieved if the control impedance contains no real component. If the control impedance has a larger real component than the terminal impedance, the efficiency is negative, i.e., the source V_{dc} must supply real power to the system.

2.3 Practical Advantages and Considerations

The switched mode synthetic admittance has a number of advantages over its linear counterpart. Some difficulties also arise that are not present in the linear case.

2.3.1. Cost Discrete power switches can be obtained for a fraction of the cost of HV linear components.

2.3.2. Size / Density The switching circuit shown in Figure 3 does not dissipate any real or reactive power flowing between the source and the controlling impedance. There is also no requirement for quiescent or bias current. Coupled with the small physical size of power switches, a low heat dissipation allows the circuit to be manufactured in an extremely small enclosure. Another significant factor is the size of the power supply. In the linear case, a large supply is required to power the components and to supply reactive power to the structure. As we have seen, in the switching case, not only is the power supply small, but if the synthesized terminal impedance has a larger real component than the controlling impedance, no power supply is required at all. The reverse is infact true, the circuit itself becomes a high voltage power supply.

2.3.3. Control Conditioning The switched mode synthetic admittance manipulates the terminal current by controlling the average voltage across a control impedance Z_c, In practice, the problem must be conditioned so that the expected current range results in realizable voltage differences across the control impedance. We can derive the voltage conditioning ratio, $\frac{V_{pwm}(s)}{V_T(s)} = 1 - \frac{Z_c(s)}{Z_T(s)}$. At a specific frequency, the problem is easily conditioned by ensuring $|Z_c(s)| >> |Z_T(s)|$, i.e., by choosing a control impedance much greater in magnitude than $Z_T(s)$. Another simple technique is to design $Z_c(s)$ having an opposite or significantly different phase angle with respect to $Z_T(s)$. In the boost configuration, we are limited in choice to an inductor and resistor.

The impedance of passive shunt damping circuits is typically comprised of inductive resistive branches. In the active frequency range, the reactance of each branch is heavily dominated by the inductor, this is expected as resonant circuits operate at very low power factors (implying small real impedance).

We must consider a number of factors: For efficiency we wish to keep the control resistance R_c small. If R_c is small, the only way to increase the control impedance, is to increase the size of the inductance L_c. As both the control and terminal impedance have a similar impedance angle (approximately $+\pi$), we cannot improve the control conditioning by relying on a phase difference. Thus, to obtain a well conditioned voltage drop across the control impedance Z_T, the control inductance must be a reasonable fraction of the terminal inductance. e.g., $L_c = \frac{L_T}{10}$.

Multimode shunt circuits include at least one inductance per branch, in this case, we must consider the lowest frequency branch, (the branch with the greatest inductance). All higher frequency branches will have an improved condition ratio.

2.3.4. Common Mode Instrumentation Performance The operation of the circuit requires the return terminals of the PZT and V_{dc} to be electrically isolated. Preferably, the acquisition of v_T should be performed using a circuit completely isolated from both references. As this is impossible in practice, the instrumentation amplifier must have a high common mode rejection ratio to attenuate components resulting from the varying potential between the two references.

3. POWER HARVESTING

As we have mentioned, the switched mode synthetic admittance is capable of absorbing energy from an electrical source.

According to (Fleming and Moheimani, 2002), the damped system transfer function from an applied actuator voltage to the measured output V_z, is

$$G_{v_z v} = \frac{V_z(s)}{V_a(s)} = \frac{K(s)G_{vv}(s)}{1 + K(s)G_{vv}(s)} \qquad (11)$$

where $K(s)$ is defined in Section 1.2. Given the damped terminal voltage (11), and the operating efficiency (10), we can quantify the harvested real power. At a specific frequency, the real power dissipated by the terminal impedance is,

$$P_T(j\omega) = \frac{1}{2}\left|V_Z(j\omega)\right|^2 \mathrm{Re}\left\{\frac{1}{Z_T(j\omega)}\right\} \qquad (12)$$

thus,

$$\begin{aligned} P_{V_{dc}}(j\omega) &= \eta\frac{1}{2}\left|\frac{V_z(j\omega)}{Z_T(j\omega)}\right|^2 \mathrm{Re}\left\{Z_T(j\omega)\right\} \qquad (13) \\ &= \frac{1}{2}\left|\frac{V_z(j\omega)}{Z_T(j\omega)}\right|^2 \left[\mathrm{Re}\left\{Z_T(j\omega) - Z_c(j\omega)\right\}\right] \end{aligned}$$

where $V_z(s) = G_{v_z v}(s)V_a(s)$, and η denotes $\eta(Z_c, Z_T, j\omega)$.

4. EXPERIMENTAL RESULTS

The switched mode synthetic admittance will now be employed to implement a two mode current blocking piezoelectric shunt damping circuit designed to damp the second and third modes of an experimental simply supported beam. In theory, the circuit is capable of harvesting power from the structure. To date, practical difficulties have avoided such operation.

The problems with power harvesting are due mainly to the highly reactive nature of piezoelectric shunt damping circuits. In the frequency range of interest, the impedance of a typical shunt circuit results in a net power flow that is 90-99 % reactive. Thus, to harvest power, the device must efficiently recycle reactive power and absorb only the minute amount of real power that would normally be dissipated by the resistance. In practice, loses due to switching, imperfect boost inductors, and other parasitic effects make such ideal operation impossible.

4.1 Experimental Setup

The experimental beam is a uniform aluminum bar with rectangular cross section and experimentally pinned boundary conditions at both ends. A pair of piezoelectric ceramic patches (PIC151) are attached symmetrically to either side of the beam surface. One patch is used as an actuator and the other as a shunting layer. Physical parameters of the experimental beam and piezoelectric transducers are summarized in Tables 1 and 2. Note that the location of the piezoelectric patch offers little control authority over the first mode. In this work, the structures second and third modes are targeted for reduction.

Length, L	0.6 m
Width, w_b	0.05 m
Thickness, h_b	0.003 m
Youngs Modulus, E_b	$65 \times 10^9\ N/m^2$
Density, ρ	2650 kg/m^2

Table 1. Experimental Beam Parameters

Length	0.070 m
Charge Constant, d_{31}	$-210 \times 10^{-12}\ m/V$
Voltage Constant, g_{31}	$-11.5 \times 10^{-3}\ Vm/N$
Coupling Coefficient, k_{31}	0.340
Capacitance, C_p	0.105 μF
Width, w_s w_a	0.025 m
Thickness, h_s h_a	$0.25 \times 10^{-3}\ m$
Youngs Modulus, E_s E_a	$63 \times 10^9\ N/m^2$

Table 2. Piezoelectric Transducer Properties

The displacement and voltage frequency responses are measured using a Polytec scanning laser vibrometer (PSV-300) and a HP spectrum analyzer (35670A).

4.2 Damping Performance

In reference (Fleming *et al.*, 2002), a piezoelectric shunt damping circuit is designed to minimize the H2 norm of the compound beam described in Section 4.1. The switched mode admittance, with a control impedance of 67 mH + 33 k- , is connected to the piezoelectric transducer and used to implement the shunt circuit.

The experimental open and closed loop transfer functions from an applied actuator voltage to the displacement at a point $G_{yv}(x = 0.17\ m, s)$ are shown in Figure 4. The amplitudes of the second and third modes are reduced by 21.6 and 21.3 dB respectively.

To analyze the linearity of the switched mode implementation, a sine wave was applied at the second mode resonance frequency, the power spectral density of the resulting voltage applied to the piezoelectric transducer is shown in Figure 5. The harmonic content and switching noise applied to the piezoelectric transducer is negligible (< 60 dB).

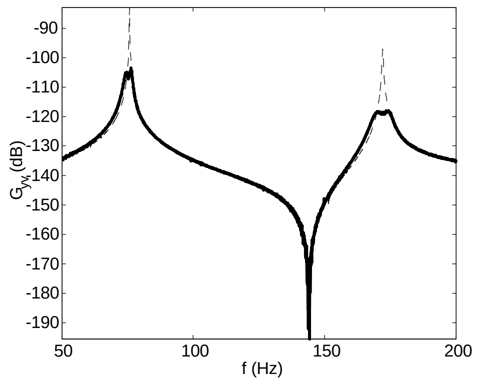

Fig. 4. Experimental open loop (- -) and damped system transfer (—) functions.

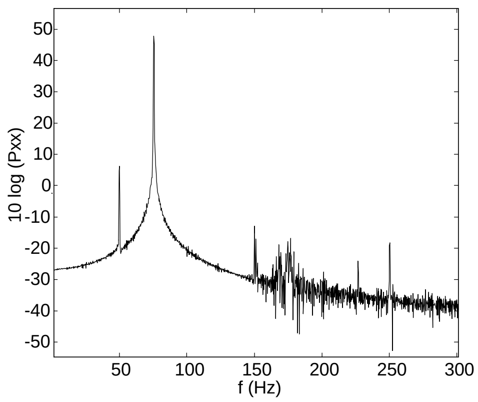

Fig. 5. Power spectral density of the terminal voltage V_z applied to the piezoelectric transducer.

5. CONCLUSIONS

Piezoelectric shunt damping circuits require impractically large inductors. A large improvement over virtual circuit implementations was achieved with the introduction of the synthetic admittance. The switched mode admittance has been presented as a low cost, high voltage, and extremely efficient alternative to its linear compliment.

Because the load is almost purely capacitive, the combination of the load and the control impedance can be designed as a low pass power filter. This allows highly linear synthesis of the voltage V_z applied to the piezoelectric transducer with negligible harmonic or switching components.

By implementing a current-flowing shunt circuit, two modes of a simply supported beam were successfully reduced in amplitude by 21.6 and 21.3 dB.

Although, ideally, the device is capable of harvesting power when implementing a circuit with non-zero resistance, the difficulties involved when attempting to synthesize a highly reactive impedance precludes such operation. Even without the ability to power harvest, the benefits of the switched mode synthetic impedance in cost, size, weight, and power efficiency make it an extremely attractive alternative for practical implementation of piezoelectric shunt damping circuits.

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