MULTIPLE MODE PASSIVE PIEZOELECTRIC SHUNT DAMPENER [1]

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Abstract: An improved method for multiple mode piezoelectric shunt damping will be presented in this paper. The proposed impedance controller has a number of benefits compared to previous shunt schemes; it is simpler to implement and requires a smaller number of passive circuit elements. The passive control strategy is validated through experimental validation on a piezoelectric laminated structure.

Keywords: Passive Compensation; Vibration Control; Piezoelctric Transducer.

1. INTRODUCTION

Piezoelectric transducers have been used as actuators and sensors for vibration control of flexible structures. Piezoelectric transducers strain when exposed to a voltage and conversely produce a voltage when strained. Placing an electric impedance across the terminals of a piezoelectric transducer, which is normally attached to or embedded in a host structure, is referred to as piezoelectric shunt damping. Piezoelectric shunt damping is regarded as a simple, low-cost, light-weight, and easy-to-implement method for controlling structural vibrations. Passive shunt damping is considered to be a stable and robust method to control structural vibration, compared to active control schemes.

Forward (Forward, 1979) experimentally demonstrated the use of resistive (R) and inductive-resistive ($L-R$) resonant piezoelectric shunt circuits. Hagood and von Flotow (Hagood and A. Von Flotow, 1991) later presented analytical models for both R and $R-L$ dampened systems. While other researchers such as (Edberg *et al.*, 1992; Hollkamp, 1994; Wu, 1998), have extended shunt damping to multiple modes using a single piezo-transducer.

Reference (Edberg *et al.*, 1992) introduced the concept of capacitive-inductive-resistive ($C-L-R$) multiple mode shunt damping, while (Hollkamp, 1994) speculated an alternative design. Although, neither of them conjectures a straightforward method for determining the required $C-L-R$ circuit parameters. Reference (Wu, 1998), introduced "current blocking" circuits, a parallel $C-L$ antiresonant circuit inside each $R-L$ shunting branch. There are a number of problems associated with this technique, the foremost being the complexity and physical size of the shunt circuit.

In this paper a new passive multiple mode shunt technique for controlling vibration of a flexible piezoelectric laminated structure is proposed. The effect of the "current flowing" shunt controller is studied theoretically and then validated experimentally on a simply supported plate. The current flowing shunt controller is similar in nature to "current blocking" proposed by (Wu, 1998), as a single piezoelectric transducer is used to dampen multiple modes. While achieving comparable performance to that of the current blocking method, it is; simpler to implement, requires fewer passive

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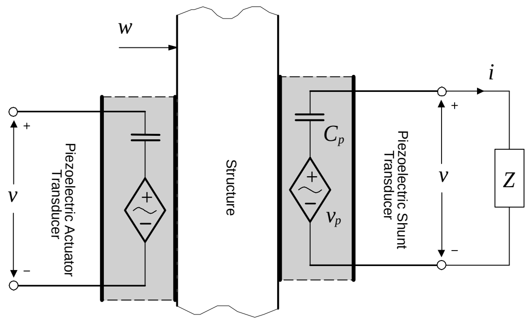

Fig. 1. A piezoelectric laminated structure shunted to an impedance Z.

elements and is less dependent on neighbouring circuit components.

The paper is organized as follows. Section 2 introduces a method for modeling a piezoelectric transducer and the composite system, i.e. the piezoelectric shunt damped system. The next section, Section 3, presents the proposed "current flowing" shunt controller. Section 4, includes experimental results that verify the proposed controller on a piezoelectric laminated simply supported plate. Finally in Section 4, the paper is concluded.

2. DYNAMIC MODELING OF LAMINATED PIEZOELECTRIC STRUCTURE

2.1 Piezoelectric Model

Piezoelectric crystals have a unique ability to convert mechanical strain into electrical energy and *vice versa*. There are many types of piezoelectric materials, namely quartz, Rochelle salt, barium titanate, lead zirconate titanate (PZT) and polyvinylflouride (PVDF). The last two materials are commonly used for the control of vibration.

For vibration control, a thin sliver of piezoelectric material is sandwiched between two conducting layers. This forms a piezoelectric transducer. The transducer is then glued to the surface of the structure using a strong adhesive material. Sometimes the piezoelectric transducer is laminated within the flexible structure.

Piezoelectric transducers behave electrically like a capacitor C_p and mechanically like a stiff spring. It is common practice to model the piezoelectric element as a capacitor C_p in series with a strain dependent voltage source v_p (Hagood and A. Von Flotow, 1991; Behrens, 2001), as shown in Figure 1. It was discussed by (Jaffe *et al.*, 1971; Fuller *et al.*, 1996) that for small strains and voltages, the piezoelectric transducer can be considered to be relatively linear.

2.2 Modeling the Composite System

When observing the dynamics of an elastic linear structure, shown in Figure 1, it is common practice to consider the transfer function between the displacement at a point on the structure and the actuator voltage $G_{wv}(s)$. Also, the dynamics between the piezoelectric shunting voltage, assuming the shunt impedance $Z(s)$ is open circuit (i.e. $Z(s) = \infty$), and the actuator voltage $G_{vv}(s)$.

In order to design an effective shunt controller it may be necessary to obtain analytical models for $G_{wv}(s)$ and $G_{vv}(s)$. Sometimes this is not possible since the flexible structure in question may be too difficult to model analytically, so a system identification method could be considered. Subspace based system identification techniques have proven to be an efficient means of identifying dynamics of high order, highly resonant systems. A full summary of the subspace based system identification technique is described in reference (McKelvey *et al.*, 2002).

Consider Figure 1, where the structure is disturbed by w and a voltage v is simultaneously applied across the terminals of the piezoelectric actuator, we may write (Behrens, 2001; Behrens *et al.*, 2001; Moheimani *et al.*, 2002)

$$v_p(s) = G_{wv}(s)w(s) - G_{vv}(s)v(s) \qquad (1)$$

assuming $Z(s) = \infty$. Hence, the current-voltage relation of the shunt impedance Z, shown in Figure 1, can be represented as

$$v(s) = i(s)Z(s), \qquad (2)$$

where v is the voltage across the impedance and i is the current flowing through the impedance Z. Applying Kirchoff's voltage law to Figure 1, we obtain the following linear relationship for v, as

$$v(s) = v_p(s) - \frac{i(s)}{C_p s}. \qquad (3)$$

Noting that C_p is the capacitance of the piezoelectric shunting element.

Manipulating equations (1), (2) and (3), the controlled displacement-voltage system $\hat{G}_{wv}(s)$ can be derived, as (Behrens, 2001; Behrens *et al.*, 2001; Moheimani *et al.*, 2002)

$$\hat{G}_{wv}(s) \triangleq \frac{G_{wv}(s)}{1 + G_{vv}(s)K(s)}. \qquad (4)$$

where K is the controller. That is,

$$K(s) = \frac{Z(s)}{Z(s) + \frac{1}{C_p s}}. \qquad (5)$$

From equation (4), it can be observed that shunt damping is in fact a feedback control problem.

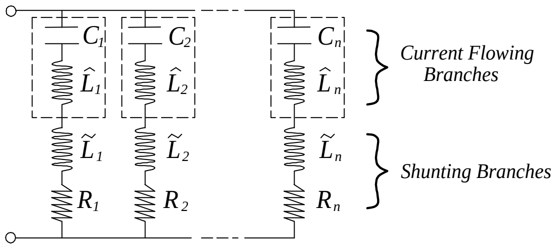

Fig. 2. Proposed "current flowing" multiple mode shunt circuit.

For a full description of the piezoelectric shunt feedback system the reader is referred to (Behrens, 2001; Behrens *et al.*, 2001; Moheimani *et al.*, 2002).

3. DEVELOPING THE "CURRENT FLOWING" SHUNT CONTROLLER

The "current flowing" shunt is similar in nature to the "current blocking" circuit (Wu, 1998). Instead of preventing the current from flowing at a specific resonance frequency ω_k $(k = 1, 2, 3, \ldots, n)$, we allow the current to flow. We do this by using a series capacitor-inductor circuit $C_k - \hat{L}_k$, shown in Figure 2. The series $C_k - \hat{L}_k$ is tuned into the resonant structural frequency ω_k, while the shunting branch $\tilde{L}_k - C_p$ is also tuned to ω_k.

Take for example, for n modes,

$$\tilde{L}_1 = \frac{1}{\omega_1^2 C_p} \quad \cdots \quad \tilde{L}_n = \frac{1}{\omega_n^2 C_p} \tag{6}$$

where $\tilde{L}_k$ is tuned into C_p. Frequencies ω_k are the resonance frequencies to be passively controlled and assuming that $\omega_1 < \omega_2 < \ldots < \omega_n$. The relationship for $\hat{L}_k$ current flowing branches is

$$\hat{L}_1 = \frac{1}{\omega_1^2 C_1} \quad \cdots \quad \hat{L}_n = \frac{1}{\omega_n^2 C_n}. \tag{7}$$

Combining (6) and (7), we obtain

$$L_1 = \frac{C_p + C_1}{\omega_1^2 C_1 C_p}$$
$$\vdots$$
$$L_n = \frac{C_p + C_n}{\omega_n^2 C_n C_p}. \tag{8}$$

By adding the inductor values together (e.g. $L_k = \tilde{L}_k + \hat{L}_k$), the total impedance for each shunt branch Z_k has been simplified. Therefore, the new or *modified* current flowing shunt, shown in Figure 3, contains one less passive element in each shunt branch. Hence, the total shunt branch impedance Z_k, is

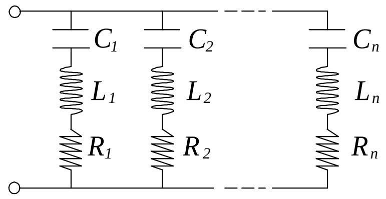

Fig. 3. Proposed modified "current flowing" multiple mode shunt circuit.

$$Z_1(s) = \frac{s^2 + \frac{R_1}{L_1}s + \frac{1}{L_1 C_1}}{\frac{1}{L_1}s}$$
$$\vdots$$
$$Z_n(s) = \frac{s^2 + \frac{R_n}{L_n}s + \frac{1}{L_n C_n}}{\frac{1}{L_n}s}, \tag{9}$$

or the admittance Y_k is

$$Y_1(s) = \frac{\frac{1}{L_1}s}{s^2 + \frac{R_1}{L_1}s + \frac{1}{L_1 C_1}}$$
$$\vdots$$
$$Y_n(s) = \frac{\frac{1}{L_n}s}{s^2 + \frac{R_n}{L_n}s + \frac{1}{L_n C_n}}. \tag{10}$$

Therefore, the total shunting admittance is

$$Y(s) = \sum_{k=1}^{n} \frac{\frac{1}{L_k}s}{s^2 + \frac{R_k}{L_k}s + \frac{1}{L_k C_k}}. \tag{11}$$

The reason for choosing to work with $Y(s)$ is that we can us the "synthetic impedance" suggested by (Fleming *et al.*, 2000) to experimentaly simulate the required passive shunt controller. The term synthetic impedance denotes a two terminal device that establishes an arbitrary relationship between voltage and current at its terminals, assuming that the admittance is stable and proper.

The reader may also note that the proposed piezoelectric shunt controller, shown in Figure 3, looks very similar to (Hollkamp, 1994), except for an absent capacitance in the first shunt branch.

4. "CURRENT FLOWING" SHUNT CONTROLLER VALIDATION

This section will validate the proposed passive shunt controller using experimental results obtained on a piezoelectric laminated bounded plate.

4.1 Boundary Supported Structure

A photograph of the piezoelectric laminate structure in question is shown in Figure 4. Two piezoelectric patches are bonded to the structure using a strong adhesive material. One piezoelectric

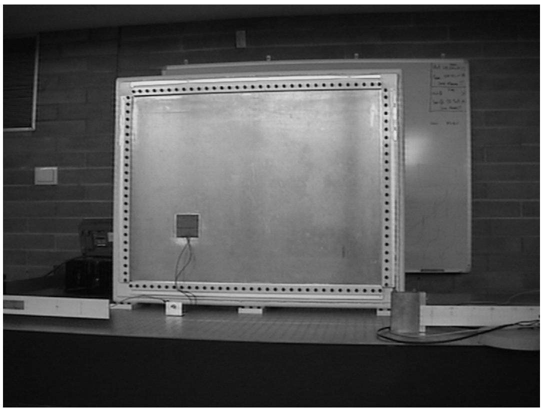

Fig. 4. Piezoelectric laminated bounded structure.

patch will be used as an actuator (which is visible in Figure 4) to generate a disturbance and the other as a shunting layer (which is not visible). For a detailed description of the apparatus, the reader is referred to (Halim and Moheimani, 2000).

Using the Polytec laser scanning vibrometer (PSV-300) and the Hewlett Packard spectrum analyzer (35670A) frequency responses were obtained for $G_{wv}(s)$ and $G_{vv}(s)$. Using the above system identification technique (McKelvey *et al.*, 2002), it can be observed that the identified model is a good representation of the true system for the first 6 modes of interest, shown in Figures 5 and 6.

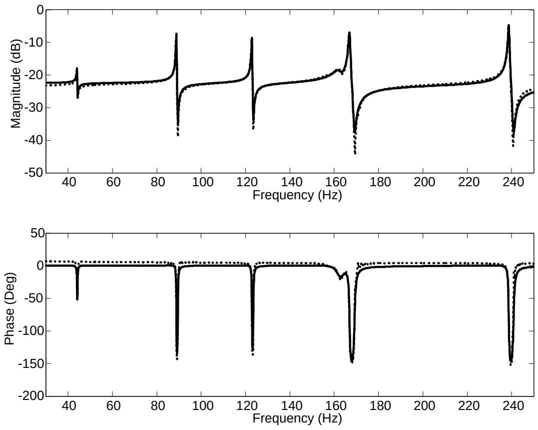

Fig. 5. Frequency response $G_{vv}(s)$, for the piezoelectric laminated bounded structure. Experimental data $(\cdots)$ and model obtained using subspace based system identification (—).

Considering, Figures 5 and 6, we can obtain the resonant modes of the structure. The resonant frequencies of the structure are $\omega_1 = 44.85Hz$, $\omega_2 = 90.2Hz$, $\omega_3 = 124.2Hz$, $\omega_4 = 161.6Hz$, $\omega_5 = 167.6Hz$ and $\omega_6 = 237.2Hz$. The 1^{st}, 2^{nd}, 3^{rd}, 5^{th} and 6^{th} structural modes were chosen for damping due to their highly resonant amplitudes. The reason for neglecting the 4^{th} mode is due to the reduced control authority.

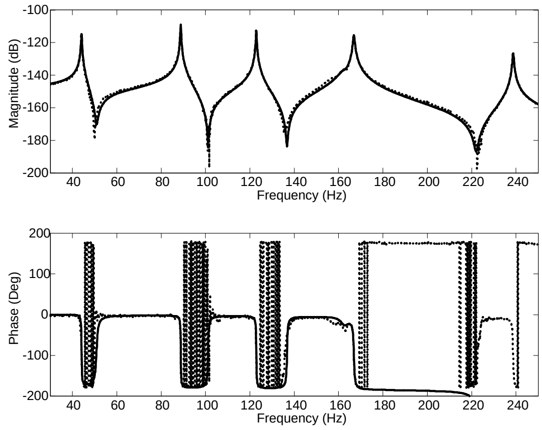

Fig. 6. Frequency response $G_{wv}(s)$, for the piezoelectric laminated bounded structure. Experimental data $(\cdots)$ and model obtained using subspace based system identification (—).

Assuming the capacitance values C_1, C_2 ,C_3, C_5 and C_6 to be $7nF$ and the experimentally measured piezoelectric shunt capacitance C_p is $67.9nF$, we can calculate the required inductance values using (8). The calculated inductor values are shown in Table 1.

To determine the appropriate shunt resistance R_k, an optimization approach could be used. An optimization technique was proposed by (Behrens and Moheimani, 2001), where the $\mathcal{H}_2$ norm of the composite system is minimized. By minimizing the $\mathcal{H}_2$ norm of $\hat{G}_{wv}(s)$ the required shunt resistance values were found to be, as displayed in Table 1.

Table 1. Circuit parameters.

Inductor	H	Resistor	Ω
L_1	1986.3	R_1	2498.2
L_2	491.1	R_2	1858.3
L_3	259.2	R_3	1272.6
L_5	142.2	R_5	1641.5
L_6	71.1	R_6	1400.1

4.2 Simulated Results

Simulated results for the $G_{wv}(s)$ and $\hat{G}_{wv}(s)$, shown in Figure 7, show that the system has been considerably damped. Table 2 summaries the simulated amplitude reduction for the 1^{st}, 2^{nd}, 3^{rd}, 5^{th} and 6^{th} structural modes.

Table 2. Amplitude reduction.

Mode	Simulated (dB)	Experimental (dB)
1	3.2	3.8
2	10.9	10.1
3	13.2	12.8
4	13.9	13.2
5	15.8	14.7

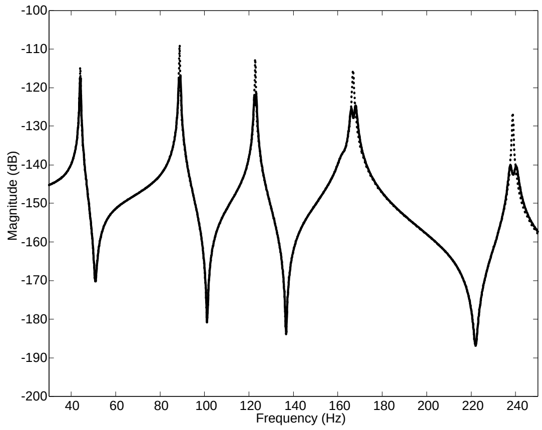

Fig. 7. Simulated frequency response: $|G_{wv}(s)|$ undamped response $(\cdots)$ and $\left|\hat{G}_{wv}(s)\right|$ damped response (—).

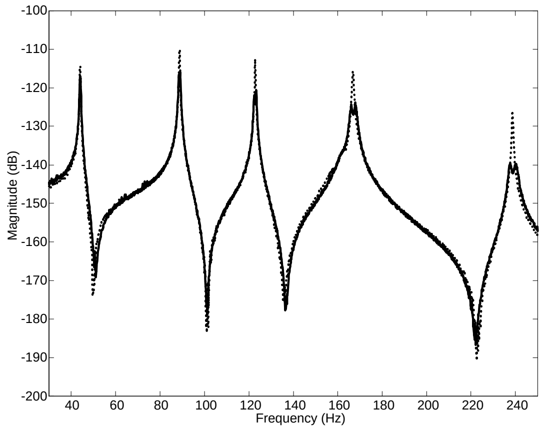

Fig. 8. Experimental frequency response: $|G_{wv}(s)|$ undamped response $(\cdots)$ and $\left|\hat{G}_{wv}(s)\right|$ damped response (—).

4.3 Experimental Results

Using the "synthetic impedance" (Fleming *et al.*, 2000) and the Polytec laser scanning vibrometer (PSV-300), we can measure $G_{wv}(s)$ and $\hat{G}_{wv}(s)$. The frequency response for the experimental undamped and damped response are shown in Figure 8. Experimental results show that the structural modes for the simply supported plate structure has been considerably damped. Table 2 summarizes the experimental dampened amplitudes.

5. CONCLUSION

A method has been presented to alleviate some of the problems associated with present passive shunt control schemes. The "current flowing" piezoelectric shunt circuit has a number of advantages compared to previous passive shunt schemes; it is simple - requires less resistors, capacitors and inductors; control selective - the controller can be applied to more dominatant modes, while neglecting lesser dominant modes; multiple mode - dampens multiple modes using a single piezoelectric transducer; and passive - it is dissipative and stable.

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