Series-Parallel Impedance Structure for Piezoelectric Vibration Damping

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ABSTRACT

This paper introduces a passive piezoelectric shunt controller, for damping multiple modes of a flexible structure using one piezoelectric transducer. The series-parallel impedance structure has a number of advantages over to previous techniques; it is simpler to implement, requires less passive elements and contains smaller inductors values. The vibration control strategy is validated through experimental work on a piezoelectric laminated cantilever structure.

Keywords: shunt impedance, vibration suppression, passive damping, multiple modes, piezoelectric transducer.

1. INTRODUCTION

When a piezoelectric laminated structure strains due to an external disturbance, a potential forms on the terminals of the piezoelectric transducer. Using an appropriately designed electrical impedance shunted across the transducer, mechanical damping can be achieved. This technique is commonly referred to as piezoelectric shunt damping.

Forward[1] demonstrated the use of resistive and inductive-resistive resonant piezoelectric shunt circuits. Hagood and von Flotow[2] later presented an analytical model for resistive and inductive-resistive shunt dampened systems. Other researchers, such as Edberg *et. al.*,[3] Hollkamp,[4] Wu[5] and Behrens *et. al.*,[6,7] have attempted to extend shunt damping to multiple modes. Authors, Hollkamp *et. al.*,[8] Sun *et. al.*,[9] von Flotow *et. al.*[10] and Fleming *et. al.*,[11] present self-tuning shunts (or adaptive shunts), where the inductor is adaptively tuned to the changing environmental conditions.

Passive piezoelectric shunts present many interesting design challenges. One such challenge is large shunt inductors required to dampen low frequency modes, it is physically impractical to implement thousands of Henries. The maximum inductor size available is 5 Henries. Consequently, active Riodan inductors[12] are synthesized using operational amplifiers. An alternative method was demonstrated by Fleming *et. al.*,[13] where a current-controlled-voltage-source and digital signal processor are used to implement the required impedance.

In this paper, we present a new class of multiple mode piezoelectric shunt damping circuit. The effect of the series-parallel impedance structure is studied theoretically and then validated experimentally on a resonant structure. The series-parallel impedance structure is similar in nature to the "current flowing" circuit as presented by Behrens *et. al.*.[6,7]

The paper is organized as follows. Section 2 introduces a method for modeling a piezoelectric transducer. Section 3 considers a piezoelectric laminated structure. Section 4, develops a method for modeling the piezoelectric damped composite system in transfer function form. The next section, Section 5, presents the proposed series-parallel impedance structure scheme. In Section 6, experimental results verify the proposed piezoelectric shunt. Finally in Section 7, the paper is concluded.

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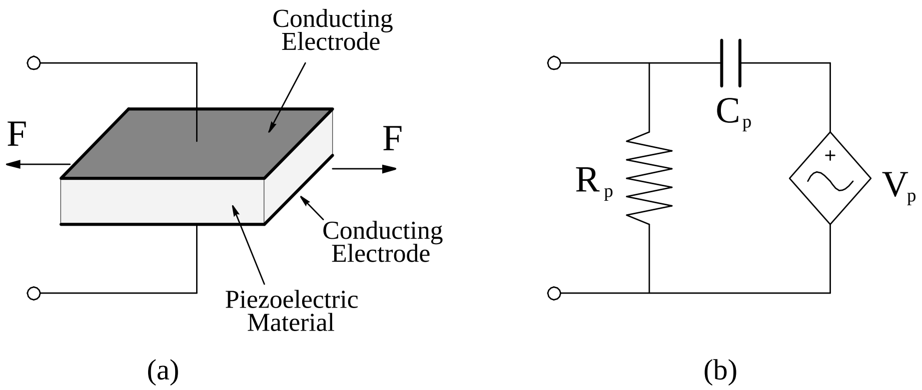

Figure 1: (a) Piezoelectric transducer and (b) lumped parameter transducer model.

2. PIEZOELECTRIC TRANSDUCER MODEL

Piezoelectricity is a phenomenon observed in certain crystals, namely quartz and Rochelle salt. However, crystals can be artificially manufactured, such as barium titanate, poly-vinylidene fluoride (PVDF) and lead zirconate titanate (PZT). Piezoelectric materials that are exposed to mechanical stress generate an electric voltage between material surfaces. This is known as the *direct* piezoelectric effect. Conversely, if the materials are subject to an electric field the crystals show mechanical deformation (*converse* piezoelectric effect). This coupling between electrical and mechanical energy makes the piezoelectric materials very useful as transducers in vibration control.

For vibration control, a thin sliver of piezoelectric material, normally lead zirconate titanate, is sandwiched between two conducting layers. This forms a piezoelectric transducer, as shown in Figure 1 (a). The transducer is then glued to the surface of the flexible structure using a strong adhesive material. Sometimes the piezoelectric transducer is laminated within the flexible structure.

When operating at low dynamic mechanical excitation i.e. $< 1kHz$, piezoelectric transducer can safely be modeled using lumped parameters, as shown in Figure 1 (b). Note that the voltage source $V_p(s)$, is proportional to the internal strain induced by the force F and the capacitance of the piezoelectric transducer C_p. Since, the parallel resistor R_p is in the range of mega-Ohms, we can assume it is approximately open circuit.

3. STRUCTURAL DYNAMICS

Consider Figure 2, where a pair of collocated piezoelectric transducers are adhered to the surface of a linear elastic structure. As the structure deforms, due to a point disturbance $W(s)$, an electric potential forms on the plates of the piezoelectric transducer. Since a potential has formed on the terminals of the piezoelectric transducer, a current flows through the impedance $Z(s)$.

To make the discussion clearer, the shunt impedance $Z(s)$ is removed from the circuit, i.e. the piezoelectric transducer is open circuited. The voltage across the terminals of the transducer is equivalent to $V_p(s)$, assuming we use the model, as depicted in Figure 1 (b). The voltage $V_p(s)$ is entirely due to the disturbance $W(s)$ acting on the structure. Therefore, the transfer function that relates the $W(s)$ to $V_p(s)$ is $G_{vw}(s)$. That is,

$$V_p(s) = G_{vw}(s)W(s). \tag{1}$$

Now, assume no disturbance is acting on the structure, i.e. $W(s) = 0$, while a voltage source $V_{in}(s)$ is attached to the terminals of the piezoelectric transducer. For this case, the $V_{in}(s)$ to $V_p(s)$ transfer function relationship is $G_{vv}(s)$. That is,

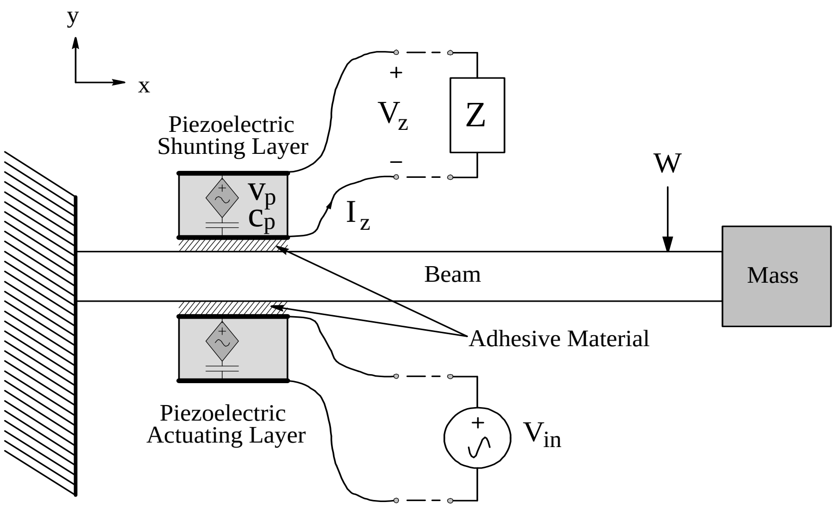

Figure 2: A piezoelectric laminated cantilever structure.

$$V_p(s) = G_{vv}(s)V_{in}(s). \tag{2}$$

Alternatively, assuming $W(s) = 0$, the displacement at a given point on the host structure $V_{in}(s)$ to $Y(s)$:

$$Y(s) = G_{yv}(s)V_{in}(s). \tag{3}$$

4. COMPOSITE SYSTEM IN TRANSFER FUNCTION FORM

In this section, we will develop a model of the piezoelectric composite system, i.e. piezoelectric laminated structure that is shunted by electrical impedance. Consider Figure 2, where a piezoelectric patch is shunted by an impedance $Z(s)$. Hence, the voltage-current relationship for the impedance can be represented in Laplace domain as

$$V_z(s) = I_z(s)Z(s), \tag{4}$$

where $V_z(s)$ is the voltage across the impedance and $I_z(s)$ is the current flowing through the impedance $Z(s)$. Using Kirchhoff's voltage law, the circuit shown in Figures 1 (b) and 2, we obtain the following relationship for $V_z(s)$, in terms of $V_p(s)$ and $I_z(s)$

$$V_z(s) = V_p(s) - \frac{1}{C_p s} I_z(s), \tag{5}$$

where $V_p(s)$ is the voltage induced from the electromechanical coupling effect and C_p represents the "ideal" capacitance of the piezoelectric transducer. Using (4) and (5) we obtain

$$V_z(s) = \frac{C_p s Z(s)}{1 + C_p s Z(s)} V_p(s). \tag{6}$$

Now, assuming that the structure is disturbed by a voltage $V_{in}(s)$, which is applied to the actuating layer and some finite impedance $Z(s)$ is shunted across the terminals of the shunting piezoelectric layer, then the overall linear relationship is[14–16]

$$V_p(s) = G_{vv}(s)V_{in}(s) - G_{vv}(s)V_z(s), \tag{7}$$

where $G_{vv}(s)$ is the open loop transfer function from $V_{in}(s)$ to $V_p(s)$.

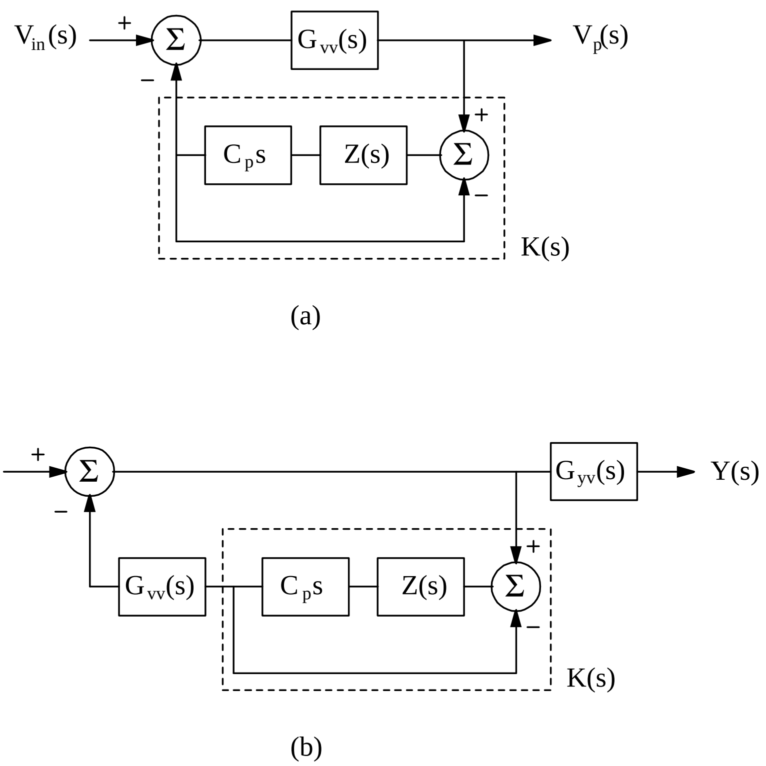

Figure 3. Composite system transfer function block diagram; (a) represents equation (8) and (b) represents equation (10).

By substituting (6) into the above equation, $V_p(s)$ is found to be

$$V_p(s) = G_{vv}(s)V_{in}(s) - G_{vv}(s)\frac{C_p s Z(s)}{1 + C_p s Z(s)}V_p(s). \tag{8}$$

Using simple algebra, the closed loop transfer function relating $V_{in}(s)$ to $V_p(s)$, can be found to be

$$\hat{G}_{vv}(s) \triangleq \frac{V_p(s)}{V_{in}(s)} = \frac{G_{vv}(s)}{1 + G_{vv}(s)K(s)} \tag{9}$$

where the effective controller $K(s)$, is

$$K(s) = \frac{C_p s Z(s)}{1 + C_p s Z(s)}. \tag{10}$$

Alternatively, it can be shown that the closed loop displacement transfer function $\hat{G}_{yv}(s)$ is[14–16]

$$\hat{G}_{yv}(s) \triangleq \frac{Y(s)}{V_{in}(s)} = \frac{G_{yv}(s)}{1 + G_{vv}(s)K(s)}. \tag{11}$$

From equations (9) - (11), it can be observed that shunt damping of piezoelectric transducer is in fact a feedback control problem, as demonstrated in Behrens *et. al.*[14–17] . A summary of the system block diagrams for both equations (9) and (11), is shown in Figure 3.

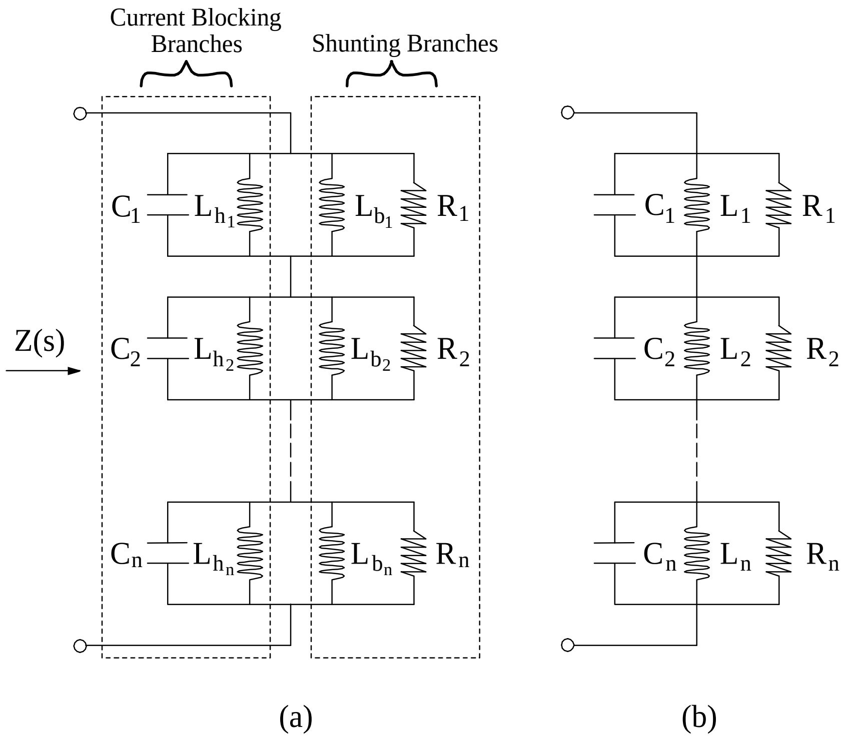

Figure 4: (a) The series-parallel impedance structure; and (b) simplified circuit.

5. DEVELOPING THE SERIES-PARALLEL IMPEDANCE STRUCTURE

In this section, a new multiple mode piezoelectric shunt damping structure is presented. The series-parallel impedance structure contains significantly smaller inductors than other resonant shunt techniques.[3, 6, 7, 18]

Consider the series-parallel impedance structure shown in Figure 4 (a). Each parallel network $C_i - L_{h_i} - L_{b_i} - R_i$ contains two sub-networks: a current blocking network $C_i - L_{h_i}$, and a parallel single mode shunt damping network $L_{b_i} - R_i$. As in reference Wu,[19] for a specific mode with resonance frequency ω_i, both the current blocking and shunt damping networks, $C_i - L_{h_i}$ and $L_{b_i} - C_p$, are tuned to ω_i.

The operation is described fairly simply. At a specific structural resonance ω_i, the current blocking network when tuned to that specific mode has an extremely large impedance. All other adjacent current blocking networks, when tuned to the remaining structural resonance frequencies, have a low impedance at ω_i. Thus, a voltage applied at the terminals results in a current that flows freely through the detuned low impedance current blocking networks and through the shunt damping network connected in parallel to the current blocking network tuned to ω_i. This way, the circuit is decoupled so that each damping network $L_{b_i} - R_i - C_p$, can be tuned individually to the target resonance frequency. At a structural resonance ω_i, the overall impedance is approximately $L_{b_i} - R_i$.

In its simplest form, as described above, the series-parallel impedance structure contains less components than traditional current blocking networks,[19] the circuit is little more than the parallel dual of so called current-flowing techniques.[6, 7] Benefits arise from a suitable choice in the arbitrary capacitances C_i. The recommended capacitance value is $10 - 20$ times larger than the piezoelectric capacitance. In this case, the current blocking inductors will be significantly smaller than the damping inductors. As shown in Figure 4 (b), the circuit can be simplified by combining the parallel current blocking and damping inductors.

When the current blocking and damping inductors are tuned to the resonance frequencies ω_i, i.e.,

$$L_{h_i} = \frac{1}{\omega_i^2 C_i} \quad \text{for all} \quad i = \{1, 2, 3, ..., n\}, \tag{12}$$

$$L_{b_i} = \frac{1}{\omega_i^2 C_p} \quad \text{for all} \quad i = \{1, 2, 3, ..., n\}. \tag{13}$$

The effective inductance resulting from the parallel connection of (12) and (13) is,

$$L_i = \left(\frac{L_{b_i} L_{h_i}}{L_{b_i} + L_{h_i}} \right) \quad \text{for all} \quad i = \{1, 2, 3, ..., n\}. \tag{14}$$

As C_i has been chosen significantly larger than C_p, we have realized a dramatic reduction in required inductance value. The impedance of the modified series-parallel impedance structure, as shown in Figure 4 (b), is

$$Z(s) = \sum_{i=1}^{n} \frac{\frac{1}{C_i} s}{s^2 + \frac{1}{R_i C_i} s + \frac{1}{L_i C_i}}. \tag{15}$$

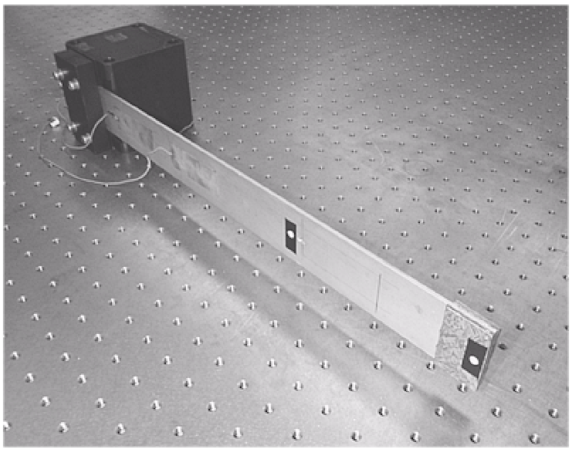

Figure 5: Experimental piezoelectric laminated cantilever apparatus.

6. SIMULATION AND EXPERIMENTAL VALIDATION

To validate the proposed series-parallel impedance structure, experiments were performed on a piezoelectric laminated cantilever structure at the Laboratory for Dynamics and Control of Smart Structures*. A photograph of the cantilever structure is shown in Figure 5. The structure consists of a uniform aluminum bar with rectangular cross section, clamped at one end. A small mass M, is attached to the free end of the structure. Two piezoelectric ceramic patches (PIC151) are attached to the surface using a strong adhesive material. One patch is used as an actuator to generate a disturbance and the other as a shunting layer, as shown in Figure 6. Dimensions of the cantilever structure and physical properties of the piezoelectric layers are summarized in Tables 1 and 2 respectively.

To present the proposed shunt controller two transfer functions need to be examined. They are, $G_{yv}(s)$ and $G_{vv}(s)$, the transfer function relating the applied actuator voltage $V_{in}(s)$ to the displacement $Y(s)$ at $x = 0.453$ m along the structure's surface; and $V_{in}(s)$ to $V_p(s)$. Since, we have identical piezoelectric transducers we can

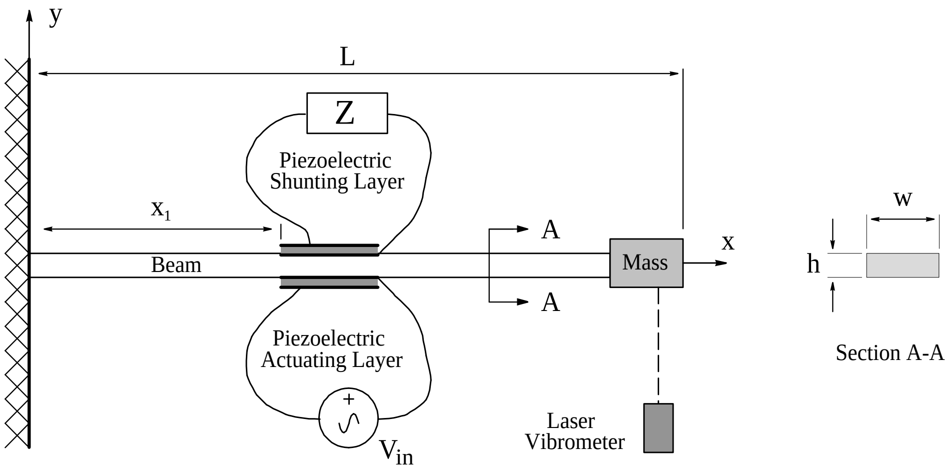

Figure 6: Experimental piezoelectric laminated cantilever structure.

Name	Symbol	Unit
Length	L	0.450 m
Width	w	0.050 m
Thickness	h	0.003 m
Young's modulus	E	65×10^9 N/m^2
Mass / unit area	ρ	10.6 kg/m^2
Mass	M	0.2 kg

Table 1: Parameters of the cantilever structure.

measure $V_p(s)$ directly. That is, by applying a voltage $V_{in}(s)$ to the actuating layer we can measure $V_p(s)$ directly, by measuring the voltage across the shunting layer.

To measure $G_{yv}(s)$ and $G_{vv}(s)$, we use a Polytec Laser Scanning Vibrometer (PSV-300) and a Hewlett Packard spectrum analyzer (35670A). In both cases, a swept sine excitation is applied to the piezoelectric actuator layer as a disturbance while measuring $V_p(s)$ and $Y(s)$ simultaneously. In our case, we have one input and two outputs system (SIMO),

$$\begin{bmatrix} V_p(s) \\ Y(s) \end{bmatrix} = G(s)V_{in}(s), \tag{16}$$

where $G(j\omega) \in \mathbb{C}^{2\times 1}$ is the open loop plant transfer function matrix.

In order to design an effective shunt controller it may be necessary to obtain an analytic model of the resonant system. Sometimes this is not possible since the flexible structure in question may be too difficult to model analytically. In such cases, a system identification method could be considered. Systems identification techniques have proven to be an efficient means of identifying dynamics of high order highly resonant systems.

In this paper we employ the algorithm of Van Overschee and De Moor,[20] a continuous time frequency domain subspace method[†]. The magnitude frequency response of $G(j\omega)$ is plotted in Figure 7. Frequency samples from 0 to 200Hz were used to identify a 6 state model for $G(j\omega)$. The magnitude frequency response of the model is

[*] http://rumi.newcastle.edu.au

[†] A Matlab implementation of this algorithm is freely available by contacting the corresponding author or by visiting http://rumi.newcastle.edu.au

overlaid on the experimental data in Figure 7. Note the 1^{st}, 2^{nd}, and 3^{rd} resonant frequencies are 7.769, 60.14 and 181.3 Hz respectively.

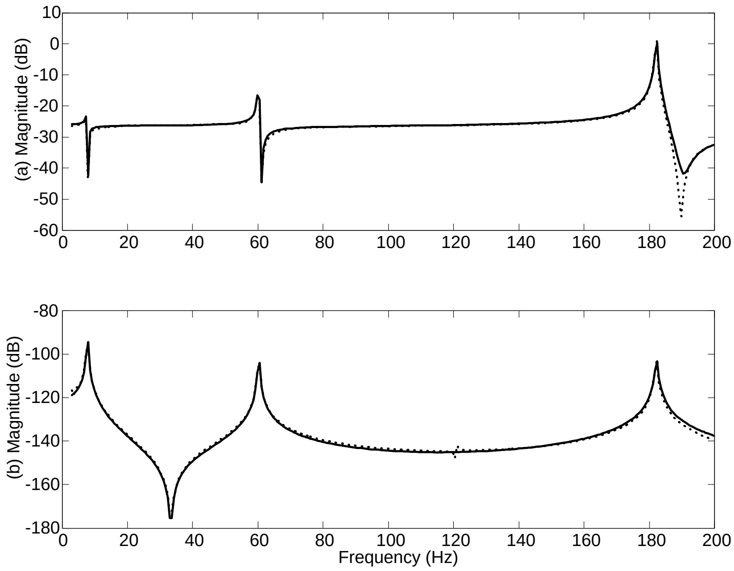

Figure 7: Frequency response of (a) $|G_{vv}(s)|$ and (b) $|G_{yv}(s)|$. Experimental ($\cdots$) and identified model (—).

A series-parallel impedance structure was designed to damp the 1^{st}, 2^{nd}, and 3^{rd} modes of the experimental apparatus. A summary of the circuit parameters is provided in Table 3. In order to determine the appropriate resistance R_n, the $\mathcal{H}_2$ norm of the $\hat{G}_{yv}(s)$ could be minimized.[21] Using the optimization strategy, as suggested by Behrens *et. al.* ,[21] the optimal resistor values were found to be as tabulated in Table 3.

To implement the required shunt impedance the synthetic impedance circuit as explained in Fleming *et. al.*[13] was used. The current controlled voltage source $v_z(t)$ is set as the output of a transfer function whose input is the current $i_z(t)$ which flows into $z(t)$, i.e. $v_z(t) = z(t)i_z(t)$. The resulting impedance as seen from the terminals is $z(t)$, as shown in Figure 8.

Name	Symbol	Unit
Actuator Location	x_1	0.130 m
Shunt Location	x_1	0.130 m
Length	L_p	0.075 m
Width	w_p	0.025 m
Thickness	h_p	0.0025 m
Capacitance	C_p	104×10^{-9} F
Young's modulus	E_p	62×10^{9} N/m^2
Strain Constant	d_{31}	-320×10^{-12} m/V
Electromechanical coupling factor	k_{31}	0.44
Stress constant / voltage coefficient	g_{31}	-9.5×10^{-3} $V\ m/N$

Table 2: Piezoelectric transducer parameters.

Symbol	Unit (H)
L_1	246.29
L_2	4.11
L_3	0.45

Symbol	Unit (μF)
C_1	1.6
C_2	1.6
C_3	1.6

Symbol	Unit ($k\Omega$)
R_1	800
R_2	150
R_3	36

Table 3: Shunt circuit parameters.

The open loop $G_{yv}(s)$ and closed loop $\hat{G}_{yv}(s)$ transfer functions, shown in Figures 9 and 10 respectively, were measured to gauge vibration damping performance. A good correlation was observed between simulated and experimental results. Peak amplitudes reduction of the 1^{st}, 2^{nd}, and 3^{rd} modes are summarized in Table 4. Therefore, we believe that the proposed piezoelectric shunt is an acceptable method for increasing mechanical damping of highly resonant structures.

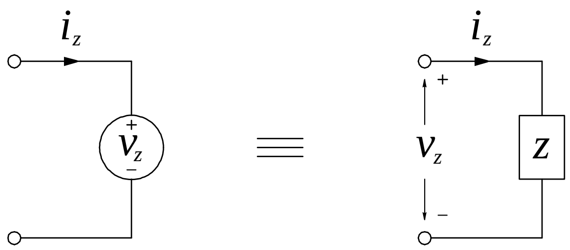

Figure 8: Functionality of the synthetic impedance.

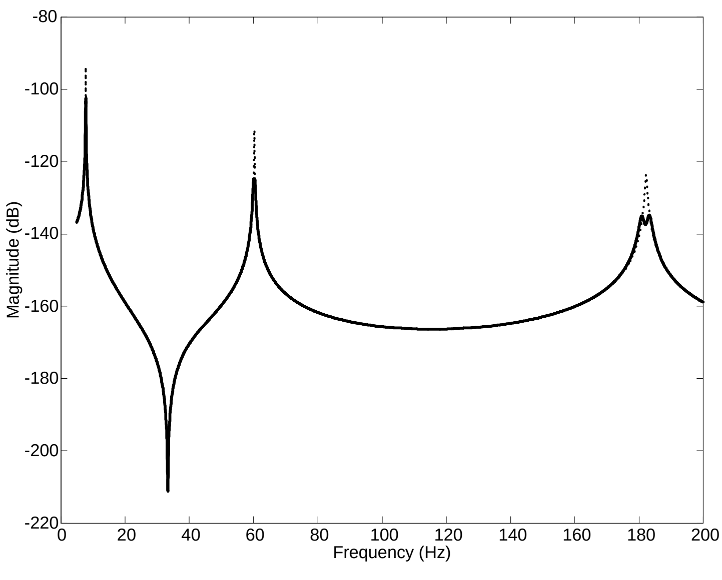

Figure 9: Simulated frequency response of $|G_{yv}(s)|$ as $(\cdots)$ and $\left|\hat{G}_{yv}(s)\right|$ as (—).

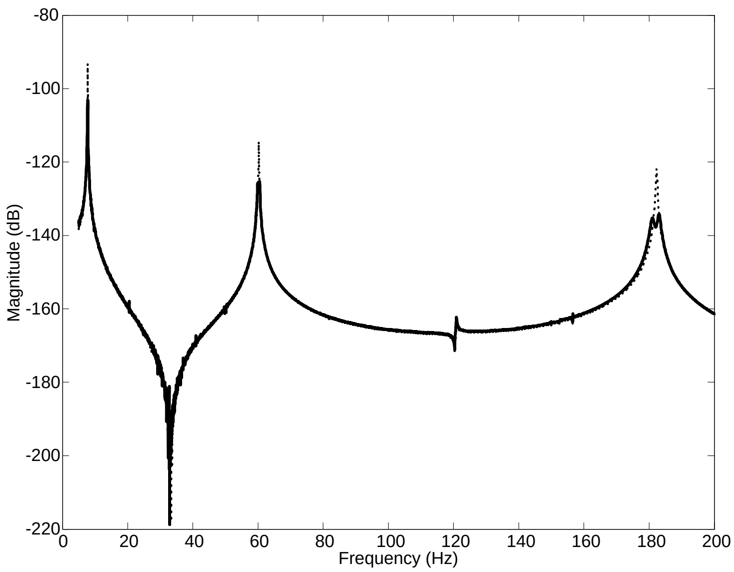

Figure 10: Experimental frequency response of $|G_{yv}(s)|$ as $(\cdots)$ and $\left|\hat{G}_{yv}(s)\right|$ as (—).

Mode	**Simulation** (dB)	**Experimental** (dB)
1	8.1	10.2
2	13.2	11.3
3	11.2	13.1

Table 4: Summary of simulated and experimental damping.

7. CONCLUSIONS

The series-parallel impedance structure has been introduced as an alternative piezoelectric shunt damping technique for reducing the vibration of multiple structural modes. While achieving comparable performance to other shunting schemes, the series-parallel impedance structure has one major advantage; smaller shunt inductor values.

The concept presented has been experimentally verified with promising results. The peak resonant amplitudes have been reduced by up to $13dB$ for three modes. In general, theoretical predictions have been coherent with experimental results.

8. ACKNOWLEDGMENTS

This research was supported by the Centre for Integrated Dynamics and Control (CIDAC) and the Australian Research Council (ARC).

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