Getting the Message About Noise Across, Loud and Clear

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Controlling the position of an actuator to within a single atom distance can be a daunting challenge. However, such demands are becoming routine in applications such as scanning probe microscopy [1], data storage [2], [3], and semiconductor manufacturing [4]. Good mechanical design and sensor choice are important; however, it is critical to have a comprehensive understanding of the noise processes involved with the system. The sensor noise, input noise, and disturbances will ultimately define the resolution and performance. Being able to quantify noise sources and predict closed-loop resolution is a valuable skill that is sometimes overlooked in undergraduate control and mechatronics education.

For a researcher involved with precision mechatronic systems, measuring and quantifying noise is often at the core of the research. For example, nanopositioning systems typically have a range in the tens of micrometers with a resolution down to fractions of an atomic diameter [5]. To further complicate matters, modern applications are demanding ever-increasing speeds. The speed at which a positioner can operate is typically limited by the sensor noise. For example, a capacitive position sensor with a range of 10 µm typically has a constant noise density of about $10\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$. The root-mean-square (rms) magnitude of the sensor noise can be found by multiplying by the effective bandwidth. For example, with a bandwidth of 100 Hz, the rms sensor noise is $10\,(\mathrm{pm}/\sqrt{\mathrm{Hz}}) \times \sqrt{100\,\mathrm{Hz}} = 100\,\mathrm{pm} = 0.1\,\mathrm{nm}$. For normally distributed noise, the peak-to-peak noise, or resolution, will be 0.6 nm, or roughly four carbon atoms. Since the magnitude of the noise is proportional to the square root of the bandwidth, doubling the bandwidth decreases the resolution by $\sqrt{2}$, which is undesirable if the objective is to move toward atomic-scale position measurement. To keep up with the simultaneous but competing demands for higher bandwidth and resolution, control engineers have turned to techniques such as new sensor and damping technologies [6] and model-based feedforward control [7].

To precisely define performance metrics such as "resolution" and to understand the tradeoff with bandwidth, a

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Linking the Noise Density, Bandwidth, and Resolution

The relationship between noise density and resolution can be illustrated by considering a sensor with a constant noise density, that is,

$$D_{yy}(f) = A_y,$$

where $D_{yy}(f)$ is the spectral density and A_y is the constant noise density of a sensor output y. The rms value of y can be found by applying the Wiener–Khinchin relation,

$$\sigma_y = A_y \sqrt{f_{bw}},$$

where σ_y is the rms value and f_{bw} is the bandwidth. If the noise is Gaussian distributed, the peak-to-peak value of the noise is approximately $6\sigma_y$. Therefore, the minimum distance between two nonoverlapping points, or the resolution δ_y, is given by

$$\delta_y = 6A_y \sqrt{f_{bw}}.$$

Thus, the sensor resolution is proportional to the noise density and the square root of the bandwidth. A more in-depth study would consider a nonconstant noise density and realistic low-pass filtering. The definition of resolution is illustrated in Figure S1 by plotting the minimum distance between three nonoverlapping points.

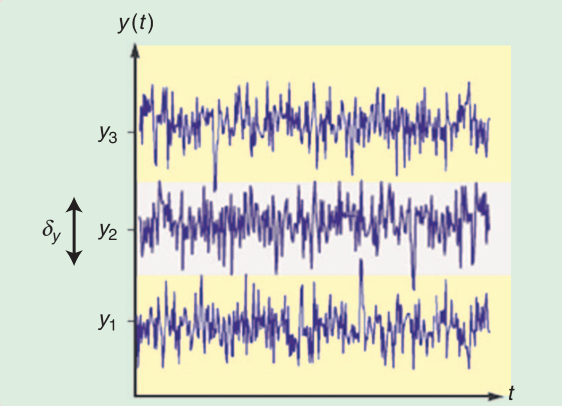

FIGURE S1. The definition of resolution is shown by plotting the minimum distance between three nonoverlapping points.

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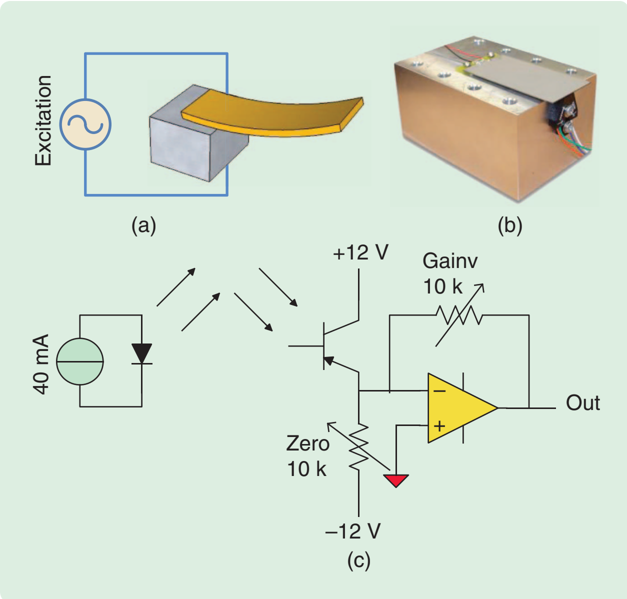


FIGURE 1 The piezoelectric bender is a two-layer brass reinforced bender from Piezo Systems, Inc. (Q220-A4-503YB). The bender has a range of $\pm$2.5 mm and can be fixed directly onto a mounting platform. The position is measured with an OPB703 reflective sensor. When driven with a 40 mA current, the phototransistor current is proportional to the distance over a range of approximately 2.5 mm (centered at 2.5 mm). The op-amp is a photocurrent amplifier with a variable gain up to 10 V/mA. The zero potentiometer is adjusted so that the output is 0 V with zero deflection.

working knowledge of random processes and the effect of on linear systems is required. Teaching these subjects can be challenging as the fundamentals require some elements from probability theory and statistics that engineering students may find somewhat dry or abstract. However, once the notion of a random process has been established, a noise source can be treated like any other signal to be manipulated, filtered, and analyzed. In addition to simulations, laboratory experiments can be utilized to gain a working knowledge of important concepts such as power spectral density and the tradeoff between resolution and bandwidth. (See "Linking the Noise Density, Bandwidth, and Resolution.")

The applications of random processes are not limited to precision mechatronics. Any in-depth study of a control application should include an understanding of the random processes that are involved. Applications where random processes are especially important include engines, flight, and chemical processes. In such applications the notion of resolution may be quantified by such metrics as the variation of an autopilot altitude controller or the predicted variance in the concentration of a chemical product. In industrial applications, the close relationship between random variance and product quality makes the ability to handle random processes a highly valued skill.

In the following, a problem-based teaching module is described with a combined laboratory experiment. A suitable experimental apparatus for this purpose is the two-layer piezoelectric bender from Piezosystems Inc. (Q220-A4-503YB), as illustrated in Figure 1. This apparatus is simple to construct and has proven to be a successful educational aid [8]. The bender has a simple second-order approximate model while being able to demonstrate higher order and nonlinear dynamics for advanced students. The frequency response is plotted in Figure 2(a). Students may be surprised by the large range of motion and fast response of the piezoelectric actuator. The direct applications of piezoelectric benders include weaving machines, laser beam scanners, Braille machines, pumps, and switches.

To provide an introduction to random processes, the learning goal is to characterize the sensor noise and to predict the closed-loop resolution of the bender. After covering the perquisites, the first laboratory exercise is to fix the

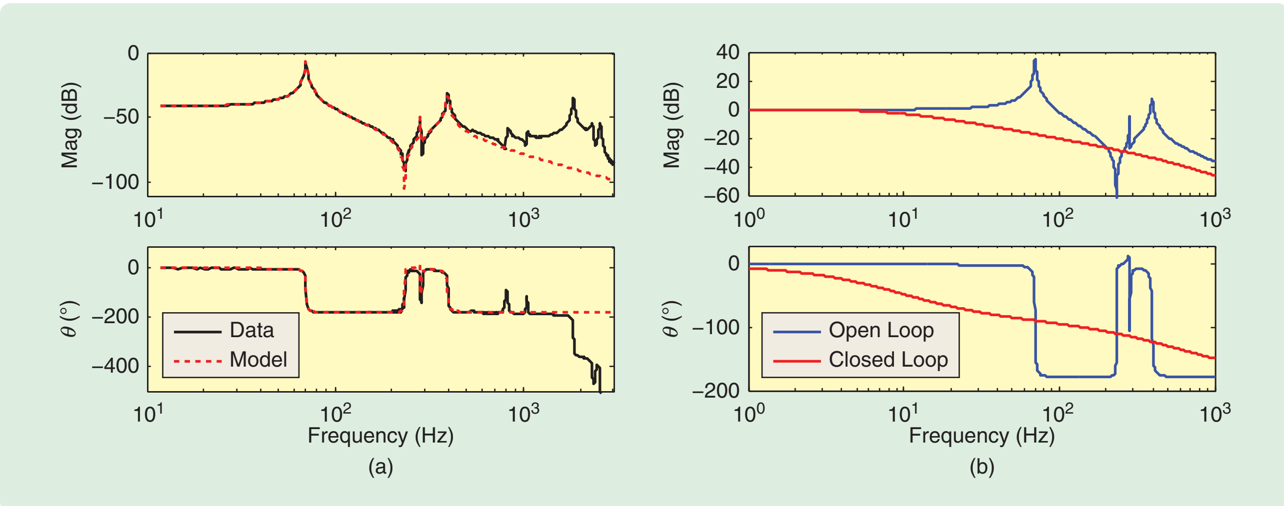


FIGURE 2 The identified model is observed to adequately represent the first three modes of the bender in (a). The closed-loop response in (b) exhibits a bandwidth of 9.5 Hz. Since the controller contains an inverse model, the resonances do not appear in the closed-loop response.

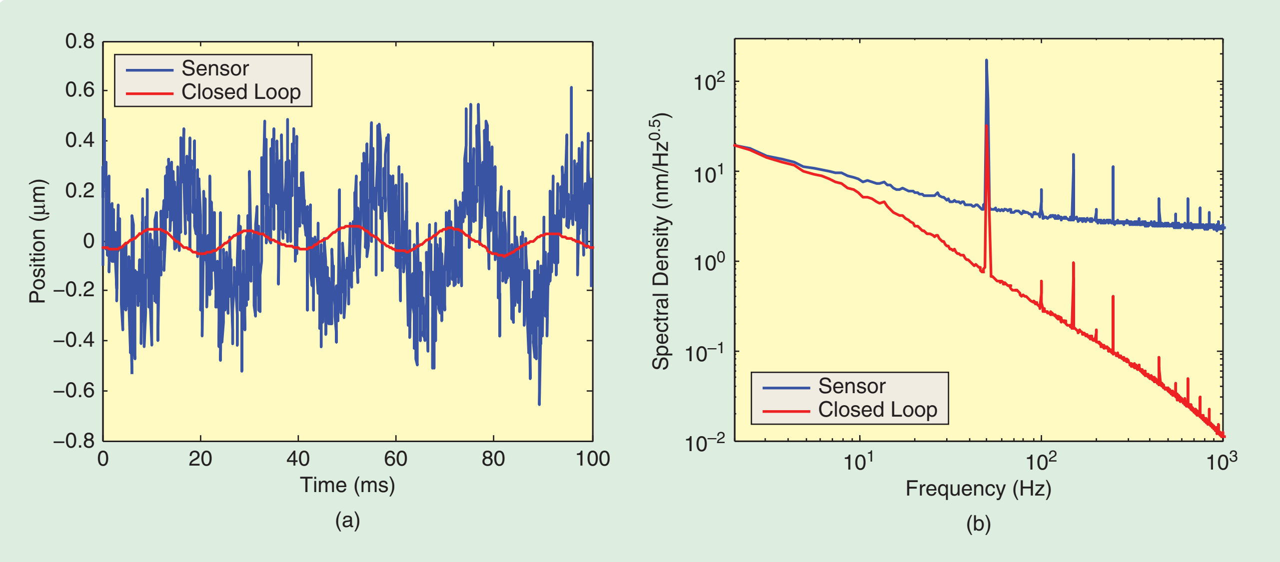


FIGURE 3 The time- and frequency-domain representations of the open-loop sensor noise and closed-loop position noise. Due to the low closed-loop bandwidth, the majority of thermal noise is filtered by the control loop. The peak-to-peak noise and closed-loop resolution is approximately 0.2 μm. An obvious extension is to design a controller that attenuates the 50 Hz interference.

bender position and record the sensor noise. To increase low level noise to a recordable level (~1 V rms), a preamplifier is required. (See "Noise Preamplifier Circuit.") The sensor noise is plotted in Figure 3(a) and exhibits a deterministic and a random component, which is typical of real-world noise sources. The Welch method for power spectrum estimation [9] is then used to determine the spectral density plotted in Figure 3(b). The periodic component can be attributed to the infrared interference emitted from fluorescent lighting (confirmed by turning the lights off!). The random component is due to thermal and photonic noise that is approximately $24\,\mathrm{nm}/\sqrt{\mathrm{Hz}}$. At frequencies below 100 Hz, the power spectrum is dominated by a $1/f$ characteristic (where f is the frequency).

Noise Preamplifier Circuit

The circuit in Figure S2 is useful for preamplifying low-level noise. It is an ac-coupled amplifier with a 0.03-Hz high-pass filter, a gain of 100, and a bandwidth of 220 kHz. All of the resistors are metal film or thin film. This circuit is powered by two 9-V batteries and mounted inside a shielded enclosure.

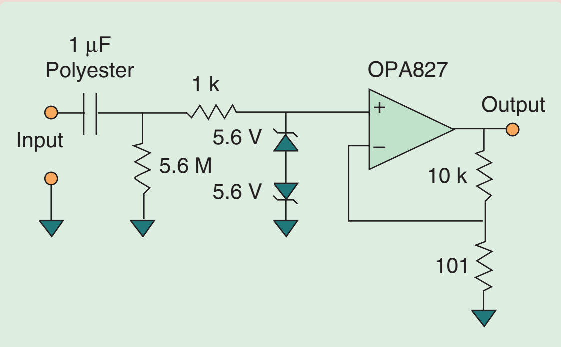


FIGURE S2. Preamplifying low-level noise.

The next task is to model the system and design a controller. A model was identified using the continuous-time frequency-domain least-squares technique (by running invfreqs.m in Matlab), which is a satisfactory approximation of the first three modes. The frequency responses of the system and model are plotted in Figure 2(a).

Since the model of this system contains two pairs of left-half plane zeros, a simple controller is the model inverse concatenated with an integrator and an additional pole for causality; that is,

$$C(s) = \frac{\alpha}{s}\frac{1}{G(s)}\frac{w_1}{s+w_1}$$

where $G(s)$ is the nominal plant model, w_1 is an additional pole at 600 Hz, and $\alpha = 60$ is the tracking controller and closed-loop bandwidth (in rad/s). The resulting closed-loop frequency response, plotted in Figure 2(b), exhibits a bandwidth of 9.5 Hz. The gain margin is 15 dB with infinite phase margin. For systems of this type, which have lightly damped high frequency modes, it is important to evaluate the phase and gain margins numerically with frequency response data over a wide bandwidth.

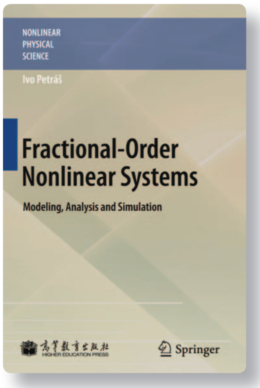
Once the closed-loop response has been derived or measured, the closed-loop position noise density can be predicted by multiplying the sensor noise density by the magnitude of the complementary sensitivity function. In

(continued on page 124)

H-infinity performance level. Furthermore, the book demonstrates that the solution of the nonfragile H-infinity filter design problem can be obtained by solving a set of linear matrix inequalities. The intended audiences are graduate students and researchers both from the fields of engineering and mathematics.

Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation

by IVO PETRÁŠ



Springer, 2011,
ISBN: 978-3-642-18100-9,
218 pages, US$139.

This book describes fractional-order chaotic systems and their analysis and numerical solution. The book introduces the fundamentals of fractional calculus, how real dynamical systems can be described using fractional derivatives and fractional differential equations, how such equations can be solved, and how to simulate and explore chaotic systems of fractional order. Simulink models for the selected fractional-order examples are also presented. The book is intended for researchers and graduate students interested in chaos phenomena or in fractional-order systems. The book can be used in graduate courses on dynamical systems, control theory, and applied mathematics.

FOCUS ON EDUCATION *(continued from page 112)*

Figure 3(b), the closed-loop position noise is observed to equal the sensor noise at low frequencies and then rolls off after the 9.5 Hz closed-loop bandwidth. The time-domain response can be observed by simulating the output of the closed-loop system in response to the sensor noise. In Figure 3(a), the closed-loop position noise is observed to be approximately 0.2 μm peak to peak. This is the smallest distance between two commanded positions that will not overlap. Many other options are available for analysis and investigation, for example, plotting the distribution functions, computing the rms value from the spectrum, and plotting the resolution versus closed-loop bandwidth. Other exercises include designing controllers that minimize the position noise, perform active noise control, and utilize feed-forward control.

Any in-depth study of a control application should include an understanding of the random processes that are involved.

These exercises are designed to introduce students to real-world noise sources and to equip them with a toolbox of techniques for quantifying and analyzing the effects of random processes on control systems.

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