Robust Speed Tracking Control of Synchronous Motors Using Immersion and Invariance

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***Abstract*—This paper presents a novel control strategy for velocity tracking of Permanent Magnet Synchronous Machines (PMSM). The model of the machine is considered within the port-Hamiltonian framework and a control is designed using concepts of immersion and invariance (I&I) recently developed in the literature. The proposed controller ensures internal stability and output regulation, and it forces integral action on non-passive outputs.**

I. Introduction

With advances in power-electronics and fast dedicated computer hardware, sophisticated control algorithms can nowadays be implemented for high-performance electrical drives based on AC machines for industrial applications. In this paper, we present a novel speed tracking control strategy for Permanent Magnet Synchronous Machines (PMSM). We write a model of the PMSM as a port-Hamiltonian system and design a robust speed regulator controller using standard methods interconnection and damping assignment—passivity-based control. We then consider a novel design to add integral action using concepts of immersion and invariance. This approach simplifies the control design, for it is only necessary to specify the target dynamics for a set of variables of dimensionality strictly less than the system original state. We then extend the results to tracking.

Port-Hamiltonian systems (PHS) are a generalization of Hamilton canonical equations of motion rooted in analytical mechanics [7], [17], [10]. PHS models offer an explicit characterisation of the interconnection structure and the energy in the system. These models are well-suited for control system design methods based on energy shaping, interconnection and damping assignment (IDA). Such control design modify the energy of the system such that the minimum of the energy is at a desired equilibrium point and at the same time damping is added to take energy out of the system; and thus, reach the desired equilibrium point [14], [6], [13], [10].

In order to add integral action to the system for robustness to low-frequency uncertainty and disturbance rejection, we follow the lead in the previous work of [1] and [16] and present a new method that uses I&I to add integral action to non-passive outputs of PHS. I&I is a methodology for designing nonlinear control systems recently proposed in [5], which proceeds by transforming the original state of the system $x(t)$ into two new states $\xi(t)$ and $z(t)$, where the dimension of $\xi(t)$ is strictly less than the dimension of $x(t)$—immersion. This new reduced state $\xi(t)$ is called the target dynamics and the transformation used to obtain these states defines a manifold. The state $z(t)$ is called off-the-manifold state and complements the dimension of $\xi(t)$. A control law is then designed to ensure that the original state is bounded, that the manifold is rendered invariant, and that the off-the-manifold state converges asymptotically to the origin. With such a control law, the original state $x(t)$ will converge to a desired equilibrium point with a dynamic behaviour that converges to that of the target dynamics $\xi(t)$. One of the advantages of I&I is that specification of desired performance is only needed for the target dynamics, which has a lower dimension than the original state.

II. PMSM as a Port-Hamiltonian System

We consider the following PHS-model of the PMSM:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-R_s & 0 & n_p x_2\\ 0 & -R_s & -n_p(x_1+\Phi)\\ -n_p x_2 & n_p(x_1+\Phi) & -b\end{bmatrix}\frac{\partial H}{\partial x} + \begin{bmatrix}v_d\\ v_q\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -T_L\end{bmatrix}, \tag{1}$$

where states are the energy variables of the system, namely, $x_1 = L_d\, i_d$, $x_2 = L_q\, i_q$, $x_3 = J\omega$, with i_d and i_q the direct and quadrature stator currents, and ω the angular speed of the rotor. The scalar $H(x) = \frac{x_1^2}{2L_d} + \frac{x_2^2}{2L_q} + \frac{x_3^2}{2J}$ is the Hamiltonian, which for this problem is the sum of the energy stored in the magnetic fields of the stator and the kinetic energy of the rotor. The controllable inputs are the direct and quadrature voltages v_d and v_q, and the disturbance is the piecewise constant load torque T_L. The stator resistance is R_s, and L_d and L_q represent the stator total inductances in the dq-frame. The constant flux of the permanent magnet is Φ and n_p is the number of poles pairs. The mechanical parameters are the linear friction b and the inertia J. When $L_d > L_q$ the model represents the salient rotor PMSM, and $L = L_d = L_q$ is the case of the round rotor PMSM [15]. The above model can also be used for control design of sine-wave brushless DC (BLDC) motors—see [11] and [8, Ch 4].

From (1), it follows that the rate of change of the energy

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stored in the system is

$$\dot{H} = [i_d, i_q, \omega]\,[v_d, v_q, -T_L]^T - \frac{\partial H}{\partial x}^T D\,\frac{\partial H}{\partial x}, \qquad (2)$$

where D is the symmetric part of the matrix that multiplies $\partial H/\partial x$ in (1). The first term in (2) is the power supplied to the system through the output-input ports and the second term represents the power dissipated by the resistors and the mechanical friction. If $D \geq 0$, then we can see that when the inputs are zero, $[v_d, v_q, -T_L] = [0, 0, 0]$, the rate of energy in the system is negative. Therefore, the energy decreases and since the Hamiltonian is bounded from below the system is stable—asymptotic stability might follow from Krasovskii-LaSalle invariance principle.

In the absence of disturbance loading torque, we can use the input voltages to design a closed-loop system that retains the PHS form such that the new Hamiltonian H_d has a minimum at $[L_d i_d^*, L_q i_q^*, J\omega^*]$, and we can add some damping so $D \to D_d$. This design in summarised in the following section.

III. Stabilisation of the Desired Equilibrium point

The control objective is to regulate the speed ω to a constant reference ω^* whilst ensuring internal stability and rejection of unknown constant load torques. The problem will be solved in two steps. In this section, we design a regulation controller to stabilise the desired equilibrium point using the classical IDA method [14]. Then, we consider integral action on the rotor speed state based on I&I. Thus, we separate the control inputs into two components: *regulation* and *integral action*, that is, $v_d = v_{dr} + v_{di}$ and $v_q = v_{qr} + v_{qi}$. We then propose the following closed-loop system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-R_1 & J_{12} & -\delta x_2\\ -J_{12} & -R_2 & -J_{23}\\ \delta x_2 & J_{23} & -B\end{bmatrix}\frac{\partial H_d}{\partial x} + \begin{bmatrix}v_{di}\\ v_{qi}\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -T_L\end{bmatrix}, \qquad (3)$$

where the desired Hamiltonian is $H_d = \frac{k_1}{2}x_1^2 + \frac{n_p\Phi}{2J_{23}L_q}x_2^2 - \frac{bx_3^*}{J_{23}J}x_2 + \frac{b}{2JB}(x_3 - x_3^*)^2$, with $\delta = \frac{n_p(L_d-L_q)}{k_1 L_d L_q}$, and the controller parameters (to be tuned) are $k_1 > 0$, $J_{23} > 0$, $R_1 > 0$, $R_2 > 0$, $B > 0$, and J_{12}. By matching equations with (1), we find that the following feedback control laws lead to the desired system (3):

$$\begin{aligned} v_{dr} =& \frac{R_s - k_1 L_d R_1}{L_d}x_1 + \frac{n_p J_{12}\Phi}{J_{23}L_q}x_2 - \frac{J_{12}b}{J_{23}J}x_3^* - \\ & \frac{np}{J}x_2 x_3 - \frac{n_p b\delta}{BJ}x_2(x_3 - x_3^*), \end{aligned} \qquad (4)$$

$$\begin{aligned} v_{qr} =& -J_{12}k_1 x_1 - \frac{n_p R_2\Phi - J_{23}R_s}{J_{23}L_q}x_2 - \\ & \frac{J_{23}b}{JB}(x_3 - x_3^*) + \frac{n_p(x_1 + \Phi)}{J}x_3 + \frac{R_2 b}{J_{23}J}x_3^*, \end{aligned} \qquad (5)$$

The closed-loop PHS (3) has a global asymptotic stable equilibrium at $x^* = (0, \frac{bL_q\omega^*}{n_p\Phi}, J\omega^*)$ when v_{di}, v_{qi}, and T_L are zero. The stability of the regulation closed loop can be directly shown by using the Hamiltonian H_d as a candidate Lyapunov function. Details of a similar regulation loop design can be found in [12], [9].

IV. Immersion and Invariance

In this section, we summarise the fundamentals of I&I approach proposed in [5], which are used in this paper.

Theorem 1 (Astolfi and Ortega, [5]): Consider the system

$$\dot{x} = f(x) + g(x)u, \qquad (6)$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. The control objective is to stabilise an equilibrium point $x^* \in \mathbb{R}^n$. Let $p < n$ and assume that the following mappings can be found:

$$\alpha : \mathbb{R}^p \to \mathbb{R}^p, \qquad \pi : \mathbb{R}^p \to \mathbb{R}^n, \qquad c : \mathbb{R}^p \to \mathbb{R}^m,$$
$$\phi : \mathbb{R}^n \to \mathbb{R}^{n-p}, \qquad \psi : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m.$$

Assume further that

A1) (Target system) The system

$$\dot{\xi} = \alpha(\xi), \qquad (7)$$

with state $\xi \in \mathbb{R}^p$, has a globally asymptotically stable (GAS) equilibrium at $\xi^* \in \mathbb{R}^p$ and $x^* = \pi(\xi^*)$.

A2) (Immersion condition) For all $\xi \in \mathbb{R}^p$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial\pi}{\partial\xi}\alpha(\xi). \qquad (8)$$

A3) (Implicit manifold) The set identity $\{x \in \mathbb{R}^n | \phi(x) = 0\} = \{x \in \mathbb{R}^n | x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^p\}$ holds.

A4) (Manifold attractivity and trajectory boundedness) All trajectories of the system

$$\dot{z} = \frac{\partial\phi}{\partial x}\Big[f(x) + g(x)\psi(x, z)\Big], \qquad (9)$$

$$\dot{x} = f(x) + g(x)\psi(x, z), \qquad (10)$$

are bounded and satisfy $\lim_{t\to\infty} z(t) = 0$.

Then, x^* is an asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f(x) + g(x)\psi(x, \phi(x)). \qquad (11)$$

Definition 1 (Astolfi and Ortega, [5]): A system $\dot{x} = f(x)+g(x)u$ is said to be I&I-stabilizable with target dynamics $\dot{\xi} = \alpha(\xi)$ if the hypotheses A1)-A4) of Theorem 1 are satisfied. The main ideas behind I&I, as well as a thorough discussion of theoretical results for stabilization and adaptive nonlinear control, have been summarized in [4].

V. Integral Control Using I&I

Using the results of the previous section, we can design an integral controller for (3). Let us define an augmented state that provides the integral action as

$$\dot{x}_4 = -k_i\frac{\partial H_d}{\partial x_3} = -\frac{k_i b}{JB}(x_3 - x_3^*). \qquad (12)$$

This additional state forces $x_3 - x_3^* = 0$ in steady state.

The fundamental problem is to solve the stabilisation of the augmented system using I&I technique. With this objective, we build a target dynamic system based on the regulation closed loop (3) and the add state (12). Indeed, from the state equations of x_1, x_3 and x_4 in (3) and (12), we propose the following target dynamics:

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_3\\ \dot{\xi}_4\end{bmatrix} = \begin{bmatrix}-R_1 & -\delta\pi_2(\xi) & 0\\ \delta\pi_2(\xi) & -B & k_i\\ 0 & -k_i & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H_t}{\partial \xi_1}\\ \frac{\partial H_t}{\partial \xi_3}\\ \frac{\partial H_t}{\partial \xi_4}\end{bmatrix} + \begin{bmatrix}0\\ -T_L\\ 0\end{bmatrix}, \quad (13)$$

where the state vector is defined as $\xi = [\xi_1, \xi_3, \xi_4]^T$, the Hamiltonian function is $H_t(\xi) = \frac{1}{2}k_1\xi_1^2 + \frac{b}{2JB}\xi_3^2 + \frac{1}{2}k_4\xi_4^2$, and k_i and k_4 are positive parameters to be chosen. The function $\pi_2(\xi)$ is a smooth function to be found. This function relates the target state ξ_4 and the original state x_2 and satisfies $x_2^* = \pi_2(\xi^*)$.

Before the design of the integral control law, we introduce two lemmas related to the stability properties of the target dynamics (13). These lemmas will be useful to prove the stability of the control system.

Lemma 1: The target dynamics (13) has an asymptotically stable equilibrium point at

$$\xi_1^* = 0, \quad \xi_3^* = 0, \quad \xi_4^* = k_4^{-1}k_i^{-1}T_L.$$

Proof: Consider the mapping $\zeta = k_i k_4 \xi_4 - T_L$, and a Hamiltonian function $H_\zeta = \frac{1}{2}k_1\xi_1^2 + \frac{b}{2JB}\xi_3^2 + \frac{1}{2}k_\zeta^{-1}\zeta^2$. Then, the target dynamics (13) can be written as

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_3\\ \dot{\zeta}\end{bmatrix} = \begin{bmatrix}-R_1 & -\delta\pi_2(\xi) & 0\\ \delta\pi_2(\xi) & -B & k_\zeta\\ 0 & -k_\zeta & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H_\zeta}{\partial \xi_1}\\ \frac{\partial H_\zeta}{\partial \xi_3}\\ \frac{\partial H_\zeta}{\partial \zeta}\end{bmatrix}, \quad (14)$$

with $k_\zeta = k_4 k_i^2$. Since k_i and k_3 are positive constant parameters, then k_ζ is positive, and thus non-singular. Using H_ζ as candidate Lyapunov function and the invariance principle, it follows that the origin is asymptotically stable. Since ξ_1, ξ_3 and ζ converge to zero, the result follows. The stability property is global since H_ζ is radially unbounded. ∎

Lemma 2: Consider the cascade interconnection of the target dynamics (13) and the system $\dot{z} = -k_z z$, with $k_z > 0$, namely,

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_3\\ \dot{\xi}_4\end{bmatrix} = \begin{bmatrix}-R_1 & -\delta\pi_2(\xi) & 0\\ \delta\pi_2(\xi) & -B & k_i\\ 0 & -k_i & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H_t}{\partial \xi_1}\\ \frac{\partial H_t}{\partial \xi_3}\\ \frac{\partial H_t}{\partial \xi_4}\end{bmatrix} - \begin{bmatrix}0\\ T_L\\ 0\end{bmatrix} + \begin{bmatrix}0\\ z\\ 0\end{bmatrix}, \quad (15)$$

Then, the trajectories of the target state ξ are bounded.

Proof: Similar to Lemma 1, we can re-write the system (15) using the mapping $\zeta = k_i k_4 \xi_4 - T_L$ and define the output y_ζ as follows

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_3\\ \dot{\zeta}\end{bmatrix} = \begin{bmatrix}-R_1 & -\delta\pi_2(\xi) & 0\\ \delta\pi_2(\xi) & -B & k_\zeta\\ 0 & -k_\zeta & 0\end{bmatrix}\begin{bmatrix}\frac{\partial H_\zeta}{\partial \xi_1}\\ \frac{\partial H_\zeta}{\partial \xi_3}\\ \frac{\partial H_\zeta}{\partial \zeta}\end{bmatrix} + \begin{bmatrix}0\\ z\\ 0\end{bmatrix} \quad (16)$$
$$y_\zeta = \frac{\partial H_\zeta}{\partial \xi_3}.$$

To prove that the trajectories of (15) are bounded we will prove GAS of the cascade system (16). We will then need to prove that (16) is smoothly dissipative and weakly zero-detectable, which ensures integral-input-to-state stability (iISS) with input z [2].

Using the storage function H_ζ and a positive constant $\epsilon < B$, it follows that

$$\begin{aligned}\dot{H}_\zeta &= -\frac{\partial H_\zeta^T}{\partial \xi_1}R_1\frac{\partial H_\zeta}{\partial \xi_1} - \frac{\partial H_\zeta^T}{\partial \xi_3}B\frac{\partial H_\zeta}{\partial \xi_3} + \frac{\partial H_\zeta^T}{\partial \xi_3}z\\ &< -\epsilon||y_\zeta||^2 + y_\zeta^T z < \frac{1}{2\epsilon}||z||^2 - \frac{\epsilon}{2}||y_\zeta||^2,\end{aligned} \quad (17)$$

which proves dissipativity of the system (16). Furthermore, (16) is weakly zero-detectable because $z = 0$ and $y_\zeta = 0$ imply $\frac{\partial H_\zeta}{\partial \xi_3} = 0$, and then from the definition of H_ζ and the dynamics (16) result $\xi_3 = 0$, $\xi_1 = 0$ and $\zeta = 0$, which prove weakly-zero detectability. Then, the system (16) with input z is iISS [2].

Since the system $\dot{z} = -k_z z$ is exponentially stable and (16) is iISS and affine in z, the cascade system is global asymptotically stable [3]. Using the inverse of the mapping, i.e., $\xi_4 = k_4^{-1}k_i^{-1}(\zeta + T_L)$, it follows that the state trajectories of (15) are bounded and they will asymptotically converge to $\xi^* = (0, 0, k_4^{-1}k_i^{-1}T_L)$. ∎

The integral control design is described in the following proposition.

Proposition 1: The PMSM control system (3) with integral action (12) is globally I&I stabilizable with control laws

$$v_{di} = -J_{12}\frac{\partial H_d}{\partial x_2}, \quad (18)$$

$$\begin{aligned}v_{qi} =& J_{12}\frac{\partial H_d}{\partial x_1} + R_2\frac{\partial H_d}{\partial x_2} + J_{23}\frac{\partial H_d}{\partial x_3}\\ &- \frac{L_q}{n_p\Phi}\Big(k_i^2 k_4\frac{\partial H_d}{\partial x_3} + k_z z\Big).\end{aligned} \quad (19)$$

The parameter $k_z > 0$ is related to the exponential convergence to zero of the off-the-manifold dynamic $z = \phi(x)$ defined as

$$z = \frac{n_p\Phi}{L_q}x_2 - k_i k_4 x_4 - \frac{b}{J}x_3^*. \quad (20)$$

The closed loop has a global asymptotically stable equilibrium at $x^* = [0, \frac{L_q(T_L + b\omega^*)}{n_p\Phi}, J\omega^*, \frac{T_L}{k_i k_4}]^T$.

Proof: To prove this proposition we will show that the proposed control law satisfies the conditions A1)-A4) of Theorem 1 with the target dynamics (13).

A1) Lemma 1 establishes that the target dynamics (13) has a global asymptotically stable equilibrium at $\xi^* = (0, 0, k_3^{-1}k_1^{-1}T_L)$. Then we look for maps $\pi(\xi) = (\pi_1, \pi_2, \pi_3, \pi_4)$ that satisfy $x^* = \pi(\xi^*)$. Given the target dynamics (13) and the control objective of regulating the rotor speed and reject constant torques, it seems natural to choose $\pi_1 = \xi_1$, $\pi_3 = \xi_3 + x_3^*$ and $\pi_4 = \xi_4$. Then $x^* = \pi(\xi^*) = (\xi_1^*, \pi_2(\xi^*), \xi_3^* + x_3^*)$. The mapping $\pi_2(\xi^*)$ has to be chosen such that satisfies the immersion condition A2).

A2) The immersion and invariance condition of Theorem 1 is satisfy if there exist mappings $\pi(\xi)$, $c_d(\pi(\xi))$ and $c_q(\pi(\xi))$ that verify:

$$\frac{J_{12}n_p\Phi}{J_{23}L_q}\pi_2 - \frac{J_{12}b}{J_{23}J}x_3^* - \delta\pi_2 k_3\xi_3 - R_1k_1\xi_1 + c_d(\pi) = \frac{\partial\pi_1}{\partial\xi}\dot{\xi},$$

$$J_{23}k_3\xi_3 - J_{12}k_1\xi_1 - \frac{n_p\Phi R_2}{J_{23}L_q}\pi_2 + \frac{R_2 b}{J_{23}J}x_3^* + c_q(\pi) = \frac{\partial\pi_2}{\partial\xi}\dot{\xi},$$

$$\delta\pi_2 k_1\xi_1 + \frac{n_p\Phi}{L_q}\pi_2 - \frac{b}{J}x_3^* = \delta\pi_2 k_1\xi_1 + k_i k_4\xi_4,$$

$$-k_i k_3\xi_3 = -k_i k_3\xi_3.$$

The third equation above leads to

$$\frac{n_p\Phi}{L_q}\pi_2(\xi) - k_i k_4\xi_4 - \frac{b}{J}x_3^* = 0, \tag{21}$$

which completes the definition of the map π. Then, the maps $c_d(\pi(\xi))$ and $c_q(\pi(\xi))$ can be found from the first two equations of the immersion condition above.

A3) From (21), we can establish the implicit form of the manifold $x = \pi(\xi)$:

$$\phi(x) = \frac{n_p\Phi}{L_q}x_2 - k_i k_4 x_4 - \frac{b}{J}x_3^*. \tag{22}$$

A4) To verify this condition, we use (22) to write the dynamic of the off-the-manifold coordinate z as

$$\dot{z} = \frac{n_p\Phi}{L_q}\Big(v_{qi} - J_{12}\frac{\partial H_d}{\partial x_1} - R_2\frac{\partial H_d}{\partial x_2} - J_{23}\frac{\partial H_d}{\partial x_3}\Big) + k_i^2 k_4\frac{\partial H_d}{\partial x_3}, \tag{23}$$

from which the control law (19) is computed such that $\dot{z} = -k_z z$. This proves that the origin of the variable z is global exponentially stable. Finally, we verify that the x-state trajectories are bounded. To do so, we use the transformation

$$\begin{aligned} \eta_1 &= x_1, \\ \eta_2 &= \phi(x), \\ \eta_3 &= x_3 - x_3^*, \\ \eta_4 &= x_4. \end{aligned} \tag{24}$$

The state equations expressed in variables η are

$$\begin{aligned}\dot{\eta}_1 =& J_{12}\frac{\partial H_d}{\partial x_2} - R_1\frac{\partial H_d}{\partial x_1} - \delta x_2\frac{\partial H_d}{\partial x_3} + v_{di} = \\ & -\delta\rho(\eta)\frac{\partial H_\eta}{\partial\eta_3} - R_1\frac{\partial H_\eta}{\partial\eta_1},\end{aligned} \tag{25}$$

$$\dot{\eta}_2 = -k_z z, \tag{26}$$

$$\begin{aligned}\dot{\eta}_3 =& \delta x_2\frac{\partial H_d}{\partial x_1} + J_{23}\frac{\partial H_d}{\partial x_2} - B\frac{\partial H_d}{\partial x_3} - T_L = \\ & \delta\rho(\eta)\frac{\partial H_\eta}{\partial\eta_1} - B\frac{\partial H_\eta}{\partial\eta_3} + k_i\frac{\partial H_\eta}{\partial\eta_4} - T_L + \\ & J_{23}\frac{\partial H_d}{\partial x_2} - k_i\frac{\partial H_\eta}{\partial\eta_4},\end{aligned} \tag{27}$$

$$\dot{\eta}_4 = -k_i\frac{\partial H_d}{\partial x_3} = -k_i\frac{\partial H_\eta}{\partial\eta_3}, \tag{28}$$

with $H_\eta = \frac{1}{2}k_1\eta_1^2 + \frac{b}{2JB}\eta_3^2 + \frac{1}{2}k_4\eta_4^2$, and the variable transformation $\rho(\eta) = \frac{L_q}{n_p\Phi}(\eta_2 + k_i k_4\eta_4 + \frac{b}{J}x_3^*)$. The control input v_{di} defined as in (18) ensure that (25) holds. The system (25)-(28) can be written as

$$\dot{\eta}_2 = -k_z z, \tag{29}$$

$$\begin{aligned}\begin{bmatrix}\dot{\eta}_1\\ \dot{\eta}_3\\ \dot{\eta}_4\end{bmatrix} =& \begin{bmatrix}-R_1 & -\delta\rho(\eta) & 0\\ \delta\rho(\eta) & -B & k_i\\ 0 & -k_i & 0\end{bmatrix}\frac{\partial H_\eta}{\partial\eta} + \\ & \begin{bmatrix}0\\ -T_L\\ 0\end{bmatrix} + \begin{bmatrix}0\\ z\\ 0\end{bmatrix}.\end{aligned} \tag{30}$$

Exponential stability of z and Lemma 2 guarantee that the trajectories η are bounded. Using the inverse of the transformation (24) the trajectories x result bounded. Then, x^* is a global asymptotically stable equilibrium of the closed loop.

■

VI. Trajectory tracking control

In the previous section, we show how the I&I-based integral control can be used to increase the robustness of the IDA-PBC of the PMSM. In this section, we apply the same technique to design a trajectory tracking controller. The control objective is to follow a time-varying speed reference. Given reference trajectory $\omega^*(t)$ and its derivative $\dot{\omega}^*(t)$ and $\ddot{\omega}^*(t)$, we propose $x_3^* = \frac{\omega^*}{J}$, $x_1^* = 0$, and

$$x_2^* = \frac{L_d L_q(J\dot{x}_3^* + cx_3^*)}{n_p J[(L_d - L_q)x_1 + L_d\Phi]}. \tag{31}$$

We define the tracking errors as $\tilde{x}_i = x_i - x_i^*$ with $i = 1, 2, 3$. Then, the error dynamics can be represented as

$$\begin{aligned}\begin{bmatrix}\dot{\tilde{x}}_1\\ \dot{\tilde{x}}_2\\ \dot{\tilde{x}}_3\end{bmatrix} =& \begin{bmatrix}-R_s & 0 & n_p\tilde{x}_2\\ 0 & -R_s & -n_p(\tilde{x}_1 + \Phi)\\ -n_p\tilde{x}_2 & n_p(\tilde{x}_1 + \Phi) & -b\end{bmatrix}\frac{\partial\tilde{H}}{\partial\tilde{x}} + \\ & \begin{bmatrix}v_d\\ v_q\\ -T_L\end{bmatrix} - \begin{bmatrix}-\frac{n_p}{J}x_2x_3^* - \frac{n_p}{J}x_2^*\tilde{x}_3\\ \dot{x}_2^* + \frac{R_s}{L_q}x_2^* + \frac{n_p(x_1+\Phi)}{J}x_3^*\\ 0\end{bmatrix},\end{aligned} \tag{32}$$

with Hamiltonian given by $\tilde{H}(\tilde{x}) = \frac{\tilde{x}_1^2}{2L_d} + \frac{\tilde{x}_2^2}{2L_q} + \frac{\tilde{x}_3^2}{2J}$. The last term in the right-hand side of (32) can be compensated using the input voltages as follows

$$v_d = \tilde{v}_d - \frac{n_p}{J}x_2x_3^* - \frac{n_p}{J}x_2^*\tilde{x}_3 \tag{33}$$

$$v_q = \tilde{v}_q + \dot{x}_2^* + \frac{R_s}{L_q} x_2^* + \frac{n_p(x_1 + \Phi)}{J} x_3^* \tag{34}$$

Performing this compensation, the tracking problem of the PMSM can be solved as a regulation problem of the compensated error dynamics using the inputs $\tilde{v}_d$ and $\tilde{v}_q$:

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \end{bmatrix} = \begin{bmatrix} -R_s & 0 & n_p \tilde{x}_2 \\ 0 & -R_s & -n_p(\tilde{x}_1 + \Phi) \\ -n_p \tilde{x}_2 & n_p(\tilde{x}_1 + \Phi) & -b \end{bmatrix} \frac{\partial \tilde{H}}{\partial \tilde{x}} + \begin{bmatrix} \tilde{v}_d \\ \tilde{v}_q \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -T_L \end{bmatrix}. \tag{35}$$

Since this system resembles the model of the PMSM (1), the tracking controller follows *mutatis mutandis* the regulation design of sections III and V. The difference is that the control law has to be computed such that the the states $\tilde{x}$ converge to asymptotically to zero. The tracking control voltages are

$$\tilde{v}_d = \frac{R_s - k_1 L_d R_1}{L_d} \tilde{x}_1 + \frac{n_p J_{12} \Phi}{J_{23} L_q} \tilde{x}_2 - \frac{np}{J} \tilde{x}_2 \tilde{x}_3 - \frac{n_p b \delta}{BJ} \tilde{x}_2 \tilde{x}_3 \tag{36}$$

$$\tilde{v}_q = \frac{R_s}{L_q} \tilde{x}_2 + \frac{n_p(\tilde{x}_1 + \Phi)}{J} \tilde{x}_3 - \frac{L_q}{n_p \Phi} \frac{k_i^2 k_4 b}{BJ} \tilde{x}_3 - \frac{L_q}{n_p \Phi} k_z z, \tag{37}$$

with $z = \frac{n_p \Phi}{L_q} \tilde{x}_2 - k_i k_4 \tilde{x}_4$. The proof of internal stability, speed tracking and rejection of piecewise unknown disturbances of the closed loop follows the same procedure than in the case of set-point regulation, then the proof is omitted.

VII. Simulation Results

In this section, we illustrate the performance of the tracking I&I-based controller designed in the previous section. We refer to this controller as I&I-track. We also simulate a classical PI nested architecture commonly used in industrial speed controllers. In this last scheme, we consider PI controllers for both direct and quadrature current, and another PI controller for the rotor velocity. We refer to this controller as PI-Control. The motor parameters are $L_d = 12\,mH$; $L_q = 6\,mH$; $R_s = 1.5\,\Omega$; $\Phi = 0.199\,v\,s$; $n_p = 2$; $J = 1.08 \times 10^{-3}\,kg\,m^2$; $b = 0.86 \times 10^{-3}\,kg\,m\,s^{-1}$ [15].

The simulation experiment consists of smooth changes of the speed references as shown in Figure 1. The motor is started unloaded and a constant torque $T_L = 2\,Nm$ at $t = 0.025$ is applied to the shaft. The sign of the load torque is changed at $t = 0.15$ as shown in Figure 7. As predicted by the theoretical results, the rotor speed converges to the time-varying reference only for the I&I-based tracking controller (see Figures 2). The proposed controller shows acceptable performance. As expected, the PI controller provides regulation of the rotor speed to the desired value when the reference is constant, however its performance is deteriorated for time-varying references (see Figure 2). Figure 7 shows that the controller state $\tilde{x}_4$ of the I&I controller compensates the load torque. Figures 3-6 show that the quadrature and direct currents as well as the input voltages are between admissible bounds and have smooth trajectories. This simulation experiment is in agreement with the theoretically-proved properties of the proposed controller in close-loop with the PMSM, namely, rotor speed-tracking with internal stability and rejection of constant unknown disturbances.

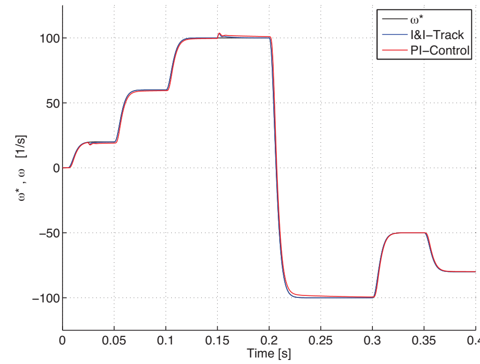

Fig. 1. Rotor speed and its reference.

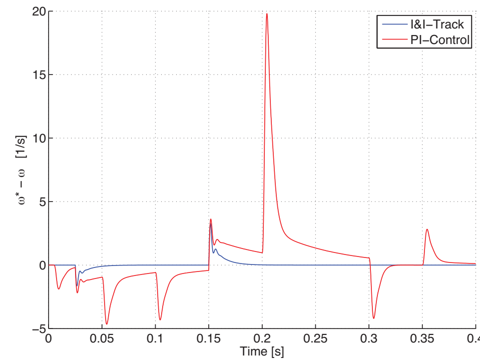

Fig. 2. Rotor speed error.

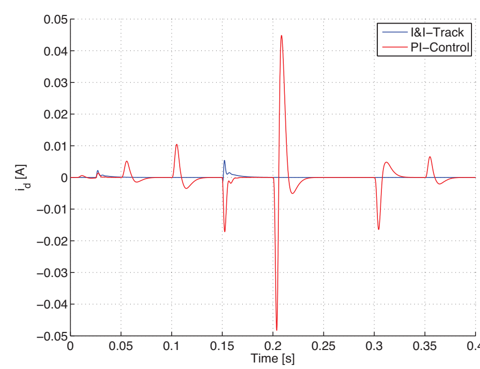

Fig. 3. Direct current.

VIII. Conclusions

In this paper, we present a method for the speed regulation and tracking control design for the PMSM that incorporate integral action to reject load torques. The design uses interconnection and damping assignment passivity based

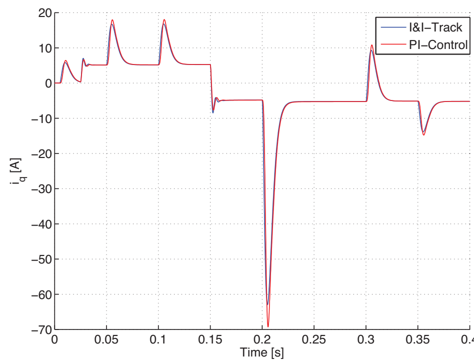

Fig. 4. Quadrature current.

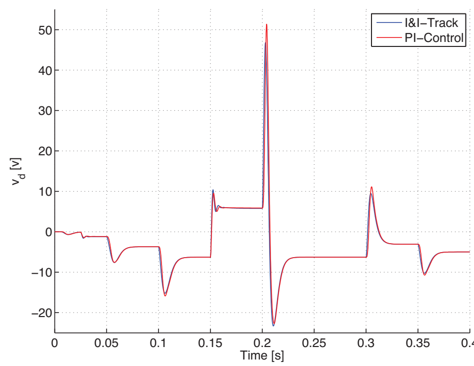

Fig. 5. Direct voltage.

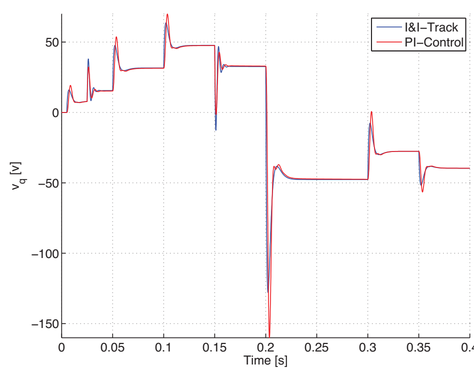

Fig. 6. Quadrature voltage.

control and immersion and invariance within the Hamiltonian framework. Using immersion and invariance theory, we prove internal stability, rotor speed regulation/tracking, and disturbance rejection. We present simulation experiment that verifies the theoretically derived properties of the I&I-based control system. The simulation are compared with the classical PI cascade controller. As expected, the tracking controller has better performance for time-varying references.

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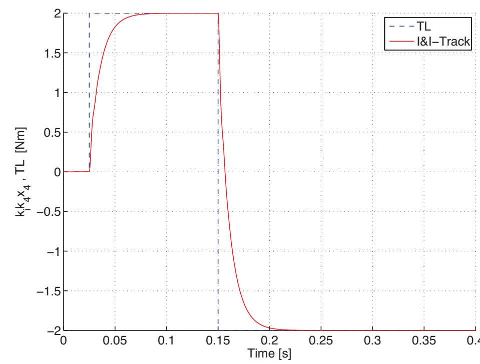

Fig. 7. Load torque and integrator state.

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