Model-free Multi-variable Learning Control of a Five Axis Nanopositioning Stage

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Abstract— This article compares the performance of recently introduced learning control methods on a 5-axis nanopositioning stage. Of these methods, the Smoothed Model-Free Inversionbased Iterative Control (SMF-IIC) method requires no modeling effort for effective tracking of repetitive trajectories and is readily applicable to multi-variable systems. Experimental results show that the tracking performance of the SMF-IIC method is similar to traditional learning control methods when applied to a single axis of the nanopositioning stage. The SMF-IIC method is also found to be effective for reference tracking of two axes simultaneously.

I. INTRODUCTION

Learning control was first introduced in the 80's to improve the tracking performance of systems with repeating reference trajectories, and has since been applied in applications, ranging from wire-bonding [1] to industrial printing [2]. The characteristics of learning control are particularly suited to challenging motion control problems, for example, nanopositioning in Atomic Force Microscopes (AFMs). Nanopositioners are high order, lightly damped systems, which makes controller design challenging and limits the achievable performance and robustness. Fortunately, AFM trajectories are pre-determined and are therefore suitable for various learning control methods. One of these learning control methods, Iterative Learning Control (ILC), has been successfully applied to nanopositioners in [3] and [4]. Another method, Repetitive Control (RC), has been investigated in [5], [6] and [7], while Inversion-based Iterative Control (IIC) has been successfully attempted in [8].

Each of these methods requires a model of the system, however, the Model-less Enhanced Inversion-based Iterative Control (M-EIIC) was developed in [9] and [10] to reduce the modeling requirements of learning control. This method is improved in [11] and extended to multi-variable systems in [12] with the Smoothed Model-Free Inversion-based Iterative Control (SMF-IIC) method.

Although the tracking performance of these methods is promising, the performance has not been directly compared to existing methods such as RC and ILC. The aim of this article

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is to provide an experimental comparison of ILC, RC, M-EIIC, and SMF-IIC applied to the challenging control problem presented by the monolithic nanopositioning stage described in [13]. This article also investigates whether the SMF-IIC method is able to effectively track a two-dimensional AFM raster scanning trajectory.

The outline of this article is as follows. In Section II the experimental setup is introduced. In Section III the SMF-IIC learning control method is discussed. The simulated and experimental performance comparison is reported in Section IV. Finally, Section V summarizes the comparison results.

II. EXPERIMENTAL SETUP



Fig. 1. Top view of the 5-axis monolithic nanopositioning stage. The stage consists of four piezoelectric benders which are connected to a fixed support on one side and to the sample table on the other.

The nanopositioning stage [13] and [14] consists of a fixed base plate, four piezoelectric bimorph actuators and a sample table, which is illustrated in Fig. 1. The working principle of the bimorph piezoelectric actuators is illustrated in Fig. 2. The four piezoelectric actuators can create motion in five degrees of freedom, three translations x, y and z and two rotations, around the x axis (θ_x) and the y axis (θ_y) respectively. Integrated sensing is realised by eight independent Tee-Rosette piezoresistive strain sensors located on the bimorph benders [14]. The output signals are normalised between [-1,1] with respect to the corresponding ranges, which are presented in Table I.

The nanopositioner is controlled by a dSpace DS1103 system, which is programmed in Simulink. Communication between MATLAB and the DS1103 system is established by

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(c) Bender performing an out-of-plane bending motion

Fig. 2. The x_{-} piezoelectric bimorph bender shown in Fig. 1 and its working principle. The bender consists of two outside layers of piezoelectric material (PZT-5A), which are polarised outwardly, with a grounded middle layer of Nickel [13], [15]. In (a), the bender is shown when no voltages are applied to either piezoelectric sheet. In (b), the bender is shown when positive voltages are applied to both piezoelectric sheets, which results in a linear motion in the positive *x* direction. In (c), the bender is shown when a positive voltage is applied to the upper sheet and a negative voltage is applied to the lower sheet, resulting in downwards bending motion of the the bimorph bender.

means of the ASAM XIL API, facilitating the system reset after each trial and offline updating of the feed forward signals, as is required for the ILC and IIC methods.

TABLE I RANGE VALUES FOR EACH OF THE FIVE AXES

Axis	Range	
x	5	$[\mu m]$
у	5	$[\mu m]$
z	16	$[\mu m]$
θ_x	500	$[\mu rad]$
θ_y	500	$[\mu rad]$

III. CONTROL METHODS

Inversion-based Iterative Control (IIC) was first introduced in [16] to compensate for errors caused by interaction during AFM scanning. An schematic overview of the IIC algorithm is shown in Fig. 3.

IIC is very similar to the commonly applied infinite time ILC method (it has been shown that both methods are equivalent under certain conditions [17]), the main difference being that the updating of the feed forward signal is performed in the frequency domain instead of using discrete filters. The main advantages of this method with respect to ILC methods are that parametric models of the plant are not necessary and stability issues resulting from inversion of non-minimum phase systems are avoided [18]. A drawback of this method is that it requires a frequency response function (FRF) model with a resolution corresponding to the length of the reference signal. The IIC



Fig. 3. Block diagram of the IIC algorithm. The system is excited with an arbitrary feed forward signal and the output is captured. The error is obtained by subtracting the reference from the output. The error is transformed into the frequency domain by a Discrete Fourier Transform (DFT) and the new feed forward signal is calculated with an update law. Finally, the new feed forward signal is transformed into the time domain by applying an Inverse Discrete Fourier Transform (IDFT) [10].

updating law was defined as

$$\begin{cases} F_j(\omega_k) = \alpha R(\omega_k) & \text{for } j = 1\\ F_{j+1}(\omega_k) = F_j(\omega_k) + L_j(\omega_k) E_j(\omega_k) & \text{for } j \ge 2 \end{cases}$$
(1)

where $\alpha \neq 0$ is pre-chosen, often as the inverse of the DC gain of the system [9], *R* is the DFT of the reference signal *r*, $L(\omega_k)$ is the learning function and *j* is the trial number. The learning function is the inverse FRF approximations of the plant, defined at normalised frequencies $\{\omega_k \in \mathbb{C} \mid \omega_k = \frac{2\pi k}{N}, k = 0, 1, ..., N - 2, N - 1\}$ [18].

When IIC was first introduced, the inverse FRF approximation had to be determined a priori. This drawback was addressed in [9] and [10], resulting in the first model-less IIC method. The rationale behind this method was to utilize input and output signals from previous trials to estimate the learning function $L_j(\omega_k) = \frac{F_j(\omega_k)}{Y_j(\omega_k)} \forall Y_j(\omega_k) \neq 0$. However, when $Y_i(i\omega_k) \rightarrow 0$ the learning function approaches infinity, resulting in an unbounded error. This major drawback was overcome in [11] by including a variable learning gain $\rho(|Y_i(j\omega_k)|) \in [0,1]$ in the updating law. The aim of this function was to reduce learning when $|Y_i(j\omega_k)|$ drops below a predefined threshold, eventually being equal to zero if $Y_i(j\omega_k) = 0$. The modelless IIC method was also extended to square multi-variable systems in [18], resulting in the SMF-IIC method. Accurately estimating a multi-variable FRF model requires $z \ge n_u$, where n_u is the number of inputs, sufficiently distinct experiments [19]. Taking this requirement into account, results in the following multi-variable learning function [18]

$$L(\boldsymbol{\omega}_k) = \mathbb{F}(\boldsymbol{\omega}_k) \mathbb{Y}(\boldsymbol{\omega}_k)^{\dagger}.$$
 (2)

 $\mathbb{F}(\omega_k)$ and $\mathbb{Y}(\omega_k)$ are defined as

$$\mathbb{F}(\boldsymbol{\omega}_k) = [F_j(\boldsymbol{\omega}_k), \dots, F_{j-z}(\boldsymbol{\omega}_k)]$$
(3a)

$$\mathbb{Y}(\boldsymbol{\omega}_k) = [Y_j(\boldsymbol{\omega}_k), \dots, Y_{j-z}(\boldsymbol{\omega}_k)]$$
(3b)

and $\mathbb{Y}(\omega_k)^{\dagger}$ is the smoothed pseudo-inverse of $\mathbb{Y}(\omega_k)$, which is defined as

$$B^{\dagger} = \sum_{i=1}^{\min(n_r, n_c)} v_i \frac{\rho(\sigma_i)}{\sigma_i} u_i^H \tag{4}$$

for an arbitrary matrix $B \in \mathbb{C}^{(n_r,n_c)}$ with singular value decomposition $B = \sum_{i=1}^{\min(n_r,n_c)} u_i \sigma_i v_i^H$. $\rho(\sigma_i)$ refers to the smoothing function, which is now a function of the singular values of the output matrix in (3b).

IV. EXPERIMENTAL RESULTS

The section compares the tracking performance of SMF-IIC to ILC, model-less enhanced IIC, and RC. Furthermore, multi-variable tracking results will also be reported for SMF-IIC.

A. Learning Control Comparison

In the first experiment, the SMF-IIC method is compared to infinite-time Iterative Learning Control (ILC) [20], Repetitive Control (RC) [21] and Model-less Enhanced Inversion-based Iterative Control (M-EIIC) [9].

The reference signal during this experiment is a single period of a 50 Hz triangle wave, which is zero phase filtered at 500 Hz to reduce excitation of the vertical resonance mode and guarantee finite acceleration. For repetitive control, this signal is repeated 20 times. For ILC, M-EIIC and SMF-IIC, the signal is zero padded to ensure that the system reaches steady state during each trial. For the experiment with the SMF-IIC method, the proposed reference signal is applied to each of the five axes. Since other methods are not readily applicable to multi-variable systems, these are only applied to the x-axis.

Each experiment is initialised with DC gain feed forward at trial number 1, which was determined *a priori* by a 10 Hz sine wave excitation. The trial errors of each method versus the iteration number is presented in Fig. 4. These results demonstrate that the tracking performance of the learning control methods after convergence is similar, generally resulting in a rms error reduction of a factor ten compared to DC gain feed forward. It is notable that the SMF-IIC method performs comparably to other methods despite being applied to five axes simultaneously and is compensating for errors caused by cross coupling. Converged time domain results of each learning control method are shown in Fig. 5. These results illustrate that the tracking error is effectively reduced to noise level.

In conclusion, the SMF-IIC method yields similar tracking performance compared to ILC, RC and M-EIIC. Advantages of the SMF-IIC method are that no prior modelling is required and that the method is readily applicable to multi-variable systems, which is not the case for ILC, RC and M-EIIC. SMF-IIC is therefore the preferable method for tracking reference signals of multi-variable systems with complex dynamics.

B. Multi-variable Learning

In the second experiment, a multi-variable reference signal is applied to the nanopositioner controlled with the SMF-IIC method. The reference for this experiment consists of two periods of a 50 Hz triangle wave, which is zero phase filtered



Fig. 4. The transient learning behaviour of the learning control methods, where the normalized rms error of the x-axis is shown as a function of the trial number.

at 500 Hz, on the x and y axis and zero reference on z, θ_x and θ_y axes. The first trial is initialised with DC gain feed forward in an equivalent manner to the first experiments. The rms error of each axis as a function of the trial number is presented in Fig. 6. Additionally, the time domain results of the range normalised error for the x and y axes at the 20th trial are shown in Fig. 7.

Fig. 6 shows that the rms error of the x and y axes is reduced by a factor of 6.6 for the x axis and 12.7 for the y axis with respect to DC gain feed forward, once convergence of the error has been achieved. This asymptotic error is bounded by 10 nm and 2.5 nm for the x and y axis respectively, as shown in Fig. 7. The SMF-IIC method is less effective in reducing the error for the z, θ_x and θ_y axes, although the asymptotic RMS error is bounded by 10 nm nonetheless.

C. Raster scanning results

For the third experiment, a raster scanning trajectory is applied to the nanopositioner, which is controlled by the SMF-IIC method. This reference consists of a staircase signal in the *x* axis, a 50 Hz triangle wave in the *y* axis and zero reference for the *z*, θ_x and θ_y axes, resulting in a 100 by 100 raster scan. This reference is zero phase filtered at 500 Hz for aforementioned reasons. The transient learning behavior resulting from this reference signal is illustrated in Fig. 8.

The tracking performance after convergence of the SMF-IIC method in this experiment is significantly worse than the previous experiment shown in Fig. 6. This is mainly caused by trial variant sensor drift, which is much more prevalent in this experiment due to the fact that the reference lasts 1.2 seconds compared to the 0.08 seconds in the second experiment. The drifting effect is illustrated in Fig. 9, where the error signal is shown at trials 14 (best case scenario), 19 (worst case scenario) and 20.

This results shows that the error mainly consists of sensor



Fig. 5. Time domain results of the learning control methods at the 20^{th} trial. The plots on the left show the normalized reference signal and the normalized output signal as a function of time. The plots on the right show the normalized error signal as a function of time.

drift and measurement noise. This illustrates that the performance of the SMF-IIC method deteriorates in the presence of a non-periodic disturbance, as is typical for all learning control methods. Such disturbances can however be effectively dealt with by a low bandwidth feedback controller.

V. CONCLUSIONS

This article demonstrates that the SMF-IIC method performs similarly to infinite time ILC, RC and model-less enhanced IIC, for control of a lightly damped five degree-of-freedom nanopositioner. This is noteworthy, as SMF-IIC requires no modelling and can be applied to multi-variable systems with little effort compared to the alternatives.

Experimental results show that the SMF-IIC method reduced the RMS error below 5 nm for axes with a non-zero reference on the experimental setup as discussed in Section II. The SMF-IIC method also improved the tracking in axes with zero reference, which cannot be achieved by SISO model-less methods.



Fig. 6. The transient learning behaviour of the SMF-IIC method resulting from a x and y triangular reference for each axis, where the normalized rms error of the each axis is provided as a function of time.



Fig. 7. Time domain results of the multi-variable learning experiment for the x and y axes. The upper plots illustrate the normalized output signal and reference signal as a function of time. The lower plots illustrate the normalized error signal as a function of time.

In the third set of experiments, a raster scan was performed, which is commonly used in applications such as atomic force microscopy. The tracking error was dominated by sensor drift which appears as a non-periodic disturbance. SMF-IIC was observed to deteriorate equivalently to other iterative methods.

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Fig. 8. The transient learning behaviour of the SMF-IIC method during the raster scanning experiment for each axis, where the normalized rms error is provided as a function of the trial number.



Fig. 9. The time domain error results of the SMF-IIC method during the raster scanning experiment for the x axis at trials 14, 19 and 20, where the normalised error of the x-axis is provided as a function of time.

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